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## DISCUSSIONS

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* Publication of closing discussion pending.

AMERICAN SOCIETY OF CIVIL ENGINEERS
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PAPERS

# CRACK PREVENTION PROGRAM, HIWASSEE DAM

By O. Laurgaard,[1] M. Am. Soc. C. E.

## Synopsis

As completed, Hiwassee Dam, a Tennessee Valley Authority (TVA) structure in western North Carolina, with a height of 322 ft from the lowest rock foundation to the roadway, was the highest overflow gravity dam. Its concrete, manufactured from the local graywacke rock, had to be placed largely during the warm summer months. To insure that the dam would be impervious to the passage of water and its surface would be weather-resistant, it was important that the mass should be free from major cracks. This required a concrete with a gradual temperature rise, one that would harden slowly, and one that would permit considerable expansion before its ultimate strength was reached.

To achieve these results the program included: (1) Use of low-heat cement; (2) a low cement content; (3) thin casting lifts; (4) long exposure periods; (5) artificial cooling for mixing water; (6) washing, rinsing, and cooling the aggregate; (7) artificial cooling of the concrete in place; (8) cleanup of horizontal joints between lifts; (9) the use of steel reinforcement; (10) diagonal keyways on bulkhead joints; and (11) curing and winter protection.

This paper shows how the program progressed and how cracking has been practically eliminated by rigid control and inspection. Actually, a saving in cement resulted, which more than offset the additional cost of the crack prevention program.

## Introduction

Hiwassee Dam (Fig. 1) is one of the important units in the comprehensive development of the Tennessee Valley. It is situated in the very westerly corner of North Carolina, adjacent to Georgia and Tennessee, on the Hiwassee River, which flows about 50 miles westerly and slightly north into the Ten-

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **July 15, 1941.**

[1] Cons. Engr., Portland, Ore. (formerly Construction Engr., Hiwassee Dam).

nessee River. This overflow structure has its spillway at El. 1,503.5 above mean sea level.[2] Average monthly temperatures ranged from a summer high of about 85° F during the working shift from 6:00 a.m. to 3:00 p.m., to a winter low of about 26° F for the shift from 3:00 p.m. to 12:00 midnight.

From the beginning of operations it was an important objective to produce a monolithic structure—one that would conform closely to the design requirements, would be impervious to the passage of water under high pressure, and would resist weathering. A strict program of concrete control therefore was necessary, on account of (1) the material of which the aggregate (including the sand) was composed; (2) the difficulty of controlling the river during construction without using separate by-pass channels or tunnels; and (3) the rather warm climate, especially in the summer time.

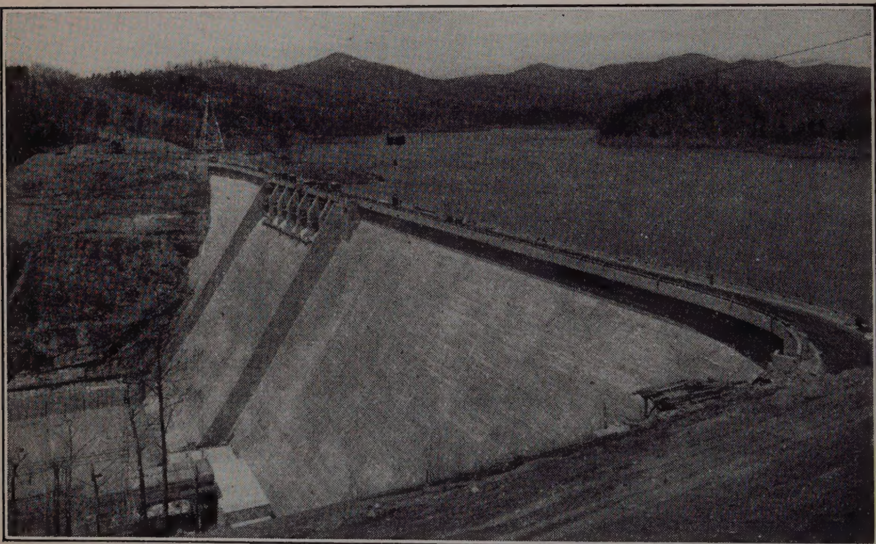

FIG. 1.—GENERAL VIEW, HIWASSEE DAM, LOOKING UPSTREAM FROM THE SOUTHWEST

Special consultants were appointed by the TVA to review conditions and submit a report. Two of these consultants, R. W. Carlson, Assoc. M. Am. Soc. C. E., and R. E. Davis, M. Am. Soc. C. E., held several conferences with engineers of the Authority and submitted a program. Quoting from their report: "To obtain water tightness the structure should not only be free from major cracks, but the concrete mix should be plastic, workable and free from water gain, and the workmanship should be such as to insure the bonding of each lift to the lift below"; and again "To insure against devastating action of weathering and erosion, the exposed faces should be of high strength concrete at later ages." The consultants summarized their conclusions and recommendations to achieve the required results in a report of April, 1938.

[2] For a general description of the project, see papers by Cecil E. Pearce, Assoc. M. Am. Soc. C. E., on "Design of Hiwassee Dam," in *Civil Engineering*, covering "Basic Considerations," June, 1940, p. 340, and "Engineering Details," July, 1940, p. 433.

By following the suggested program, Hiwassee Dam has been constructed relatively free from cracks, and capable of withstanding weathering for a long time. These results have been accomplished by careful inspection and testing of the cement, by rigid control of the concrete, by selection of proper equipment for manufacturing aggregate, mixing, placing and curing of concrete, and by cooperation between engineering and construction forces. Practically all of the work has been done by the Authority's forces.

Until recently, cracking of mass concrete was regarded as inevitable. As concrete control has developed, certain contributing factors for cracking have been eliminated and thus the mass has been greatly improved. For the recommended program submitted by the consultants, supplemented by such additional features as were believed desirable and worthy by the engineers of the Authority, a number of provisions to reduce or eliminate cracks in Hiwassee Dam were adopted: (1) The use of low-heat cement; (2) a low cement content; (3) thin casting lifts; (4) long exposure periods; (5) artificial cooling of the mixing water; (6) washing, rinsing, and cooling of the aggregate; (7) artificial cooling of the concrete in place; (8) cleanup of horizontal joints between lifts; (9) the use of steel reinforcement; (10) diagonal keyways on bulkhead joints; and (11) curing and winter protection.

None of these methods is novel or original, but the combination of all of them on one job is believed to be unique. After two years of concrete placement, there appear to be no cracks on the upstream face, none of any consequence on the downstream face, and only one small crack that can be dignified by that name in the galleries. Although a few small cracks did appear on the contraction joint faces between blocks during construction, these were unavoidable because of large differences in elevation between the tops of adjacent blocks (caused by closure procedure) at a time of great variations in temperature.

## Routine Concrete Control and Procedure

Low-heat cement was furnished by five manufacturers from six different mills, with acceptance tests by the Authority. Rigid control was exercised in the manufacture of aggregate including sand, and in the mixing and placing of concrete. Frequent screen analyses served as a basis for controlling the aggregate grading. Tests were made during the first few months of concreting to determine the effect of grinding in the mixers, which were used to correct the combined aggregate grading of the material going into the mixers and to obtain a desirable grading in the fresh-mixed concrete. All concrete aggregates except sand were thoroughly washed to remove extraneous material including fines and thus aid to maintaining a reasonably constant combined aggregate grading. Rapid moisture tests on all aggregates at the mixing plant served to control the water-cement ratio. Similarly, vigilant inspection of concrete placing in the forms assured thorough compaction and proper placement.

Other measures to insure uniformly high quality included particular attention to the cleanup of old concrete surfaces and adequate curing of all concrete. Routine tests of specimens made from daily samples served as a check on the compressive strength, and other desirable properties, of the concrete. Special tests on mixer grinding, bleeding, workability, and strength relationship for

variable water-cement ratios produced additional valuable information. Pertinent data for all the concrete mixes and test specimens are given in Table 1.

TABLE 1.—Typical Concrete Mixes—Hiwassee Dam

| Item | 6-in.[a] mass | 6-in.[a] face | 3-In. Concrete with Light Reinforcing | | | | 1½-in.[a] concrete with heavy reinforcing | ¾-in.[a] concrete, extra heavy reinforcing |
|---|---|---|---|---|---|---|---|---|
| | | | 3-in.[a] face | 3-in.[a] face | 3-in.[a] mass | 3-in.[a] mass | | |
| **(a) Pounds of Material in a Cubic-Yard Batch** | | | | | | | | |
| Aggregate: | | | | | | | | |
| Sand | 838 | 771 | 887 | 867 | 986 | 996 | 1,017 | 1,474 |
| Rock (No. 4 to ¾ in.) | 511 | 525 | 682 | 667 | 705 | 639 | 954 | 1,475 |
| Rock (¾ in. to 1½ in.) | 620 | 631 | 853 | 833 | 845 | 924 | 1,209 | .... |
| Rock (1½ in. to 3 in.) | 693 | 700 | 989 | 967 | 986 | 996 | .... | .... |
| Rock (3 in. to 6 in.) | 985 | 876 | .... | .... | .... | .... | .... | .... |
| Total aggregate | 3,647 | 3,503 | 3,411 | 3,334 | 3,522 | 3,555 | 3,180 | 2,949 |
| Water | 241 | 251 | 273 | 291 | 273 | 265 | 327 | 381 |
| Cement | 301 | 433 | 470 | 501 | 342 | 331 | 564 | 658 |
| Total batch weight | 4,189 | 4,187 | 4,154 | 4,126 | 4,137 | 4,151 | 4,071 | 3,988 |
| **(b) Compressive Strengths** | | | | | | | | |
| Field Mix: | | | | | | | | |
| 7 days | 900 | 1,713 | 1,526 | 1,530 | 700 | 700 | 1,371 | 1,480 |
| 28 days | 2,509 | 4,254 | 3,916 | 3,873 | 2,177 | 1,947 | 3,567 | 4,013 |
| 90 days | 4,078 | 6,168 | 6,101 | 6,010 | 3,435 | 4,015 | 5,493 | 6,060 |
| 6 months | 4,569 | 6,560 | 6,360 | 7,025 | 4,266 | 4,255 | ....[c] | ....[c] |
| 1 year | 4,718 | 7,218 | 6,708 | ....[c] | ....[c] | 4,405 | ....[c] | ....[c] |
| Concrete Cores: | | | | | | | | |
| 313 days | 3,620 | .... | .... | .... | .... | .... | .... | .... |
| 321 days | 3,470[b] | .... | .... | .... | .... | .... | .... | .... |
| 217 days | 4,170 | .... | .... | .... | .... | .... | .... | .... |
| 225 days | 3,560[b] | .... | .... | .... | .... | .... | .... | .... |
| 203 days | 4,450 | .... | .... | .... | .... | .... | .... | .... |
| 211 days | 3,520[b] | .... | .... | .... | .... | .... | .... | .... |
| **(c) Cement per Cubic Yard** | | | | | | | | |
| Bbl. | 0.80 | 1.15 | 1.25 | 1.33 | 0.91 | 0.88 | 1.50 | 1.75 |
| **(d) Water-Cement Ratio $W/C$** | | | | | | | | |
| By weight | 0.80 | 0.58 | 0.58 | 0.58 | 0.80 | 0.80 | 0.58 | 0.58 |

[a] Maximum size of aggregate. [b] Wet. [c] Tests not completed at these ages.

## Use of Low-Heat Cement

Contracts with several cement companies had been awarded for modified cement when it was concluded to use low-heat cement. Therefore, it was necessary to pay a premium of from 5 to 10 cents per bbl for the new material. To comply with the specifications this material was called "modified cement for summer use at Hiwassee Dam," although in reality similar specifications governed the low-heat cement used at Marshall Ford Dam and elsewhere under the United States Bureau of Reclamation. Specifications for "summer cement" were similar to the modified or "Type B" cement, except that the

tricalcium aluminate ($C_3A$) was limited to a maximum of 7%, tricalcium silicate ($C_3S$) to a maximum of 35%, dicalcium silicate ($C_2S$) to 40% minimum and 65% maximum, and tetracalcium alumino ferrite ($C_4AF$) to a maximum of 20%. The "summer cement" specifications also required the fineness to be between 1,700 and 2,200 sq cm per g as against 1,600 to 2,200 for "Type B" cement.

Strength requirements are also somewhat lower for the "summer" than for the "Type B" cement. While exact data are not available to compare concrete strengths for the two types of cement, relative results for Norris and Hiwassee concretes are shown in Table 2. Although the greater strength shown on the

TABLE 2.—Comparison of Concrete Strengths, in Lb per Sq In.

| Age at test | Face concrete, Norris[a] | Face concrete, Hiwassee[b] | Mass concrete, Hiwassee[c] |
|---|---|---|---|
| 7 days | 3,617 | 1,723 | 900 |
| 14 days | 4,646 | 2,750 | 1,438 |
| 28 days | 5,500 | 4,254 | 2,509 |
| 90 days | 7,332 | 6,168 | 4,078 |
| 6 months | 7,768 | 6,560 | 4,569 |
| 1 year | 8,609 | 7,218 | 4,718 |

[a] 1.2 bbl "Type B" (modified) cement per cu yd at *W/C* ratio of 0.57 by weight.
[b] 1.15 bbl "summer" cement, at *W/C* ratio of 0.58 by weight. [c] 0.8 bbl "summer" cement, at *W/C* ratio of 0.8 by weight.

Norris Dam concrete is due to some extent to the lower water-cement ratio (W/C), it is also known that the type of aggregate was responsible for some additional strength.

For concrete using low-heat cement it was expected that the stripping of forms in the winter time would have a deterimental effect on slowing up the work, but as a matter of fact no serious difficulty arose. The principal consideration favoring low-heat cement was the reduction in temperature rise during the heat of hydration and, consequently, all other things being equal, the corresponding reduction in cracking due to temperature changes. The advantages of insuring the continuity of the structure outweighed the obvious objections of the slight additional cost of cement, the requirement of longer curing, the reduction in early strength, and possibly the necessity for more protection against damage in winter.

## Low Cement Content of Concrete

The concrete for Hiwassee Dam and spillway was composed of the two types listed in Table 2. The consultants believed that concrete must contain sufficient cement to provide strength and to insure impermeability, but at the same time that the lower its cement content, the less the tendency would be to crack. In general, dams with a larger percentage of cement have exhibited a greater than average degree of cracking and, in many cases, excessive leakage. On the other hand, some older dams, for which the cement content was less than 0.75 bbl per cu yd, were comparatively free from major cracks and yet were exceedingly watertight. With the recent tendency toward better cement,

better gradations of aggregate, the use of drier mixes placed by vibration, and better control in all respects, it seemed reasonable to expect that a low cement constant would minimize cracking.

At Hiwassee Dam the aggregate was considered suitable for the use with lean mixes. Although more water was required for a satisfactory consistency than with other aggregates, the experiments before concreting showed that a mix containing 0.75 bbl per cu yd was plastic and workable and showed almost no water gain. The consultants recommended that a W/C ratio of not more than 0.85 should be used for mass concrete and that the cement content be not more than 0.8 nor less than 0.75 bbl per cu yd. They also recommended

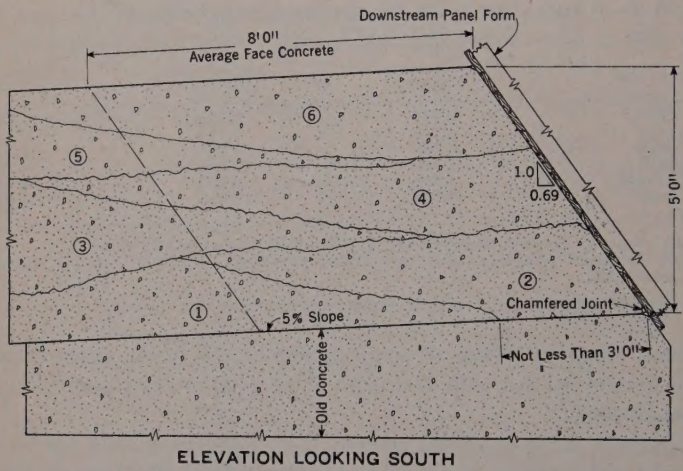


FIG. 2.—SEQUENCE OF POURS (CIRCLED NUMERALS) FOR CONCRETE ON DOWNSTREAM FACE OF DAM

that within a distance of, say, 8 ft from exposed faces, the W/C ratio should be approximately 0.6 and the cement content about 1.10 bbl per cu yd. With the W/C ratio of 0.6, using low-heat cement, a strength of 2,500 lb per sq in. was expected at the end of 28 days and a strength of 5,000 lb per sq in. at the end of a year.

After due consideration, the cement content for the mass concrete was set at 0.80 bbl per cu yd, and for the face concrete at 1.15 bbl with W/C ratios of 0.80 and 0.58, respectively. Near both faces of the dam the concrete was placed in sequence between the mass concrete and the face concrete (Fig. 2) so that no well-defined joint existed between the two types. For the reinforced sections, concretes with maximum aggregates of 3 in. and of $1\frac{1}{2}$ in. were used with a cement content of 1.33 and 1.50 bbl per cu yd, respectively, and a W/C ratio of 0.58. For mass concrete a slump of one inch or less was maintained; for face concrete, $1\frac{1}{2}$ to 2 in.; and for both types of reinforced concrete, 2 to 3 in. Typical data for one class of concrete obtained from 17-in. drilled cores are given in Table 3. The dam was constructed using blocks between vertical expansion joints which were, in general, 50 ft apart (less in the spillway section). Numbering started with block 0 at the north (right) abutment, extending to

block 8 next to the spillway; blocks 9 to 15 in the spillway section; block 16 near the power house and so on to the extreme south bank at block 27.

TABLE 3.—Test Results from Concrete Drill Cores, by U. S. Bureau of Reclamation
(Mass Concrete with 6-In. Maximum Aggregate; 0.8 Bbl Cement per Cu Yd; W/C, 0.8)

| Block No. | Hole No. | Age (days) | Strength, in Lb per Sq In.[a] | | Elastic modulus, in $10^6$ | Poisson's ratio | Unit weight,[b] in lb per cu ft |
|---|---|---|---|---|---|---|---|
| | | | Dry | Wet | | | |
| 6 | A | 313 | 3,620 | | 2.7 | 0.13 | 155.74 |
| 6 | B | 321 | | 3,470 | 2.6 | 0.20 | 154.94 |
| 16 | A | 217 | 4,170 | | 2.9 | 0.13 | 154.85 |
| 16 | B | 225 | | 3,560 | 2.6 | 0.18 | 156.75 |
| 23 | A | 203 | 4,450 | | 2.7 | 0.13 | 155.93 |
| 23 | B | 211 | | 3,520 | 2.6 | 0.13 | 155.68 |

[a] Strength corrected for H/D (ratio of height to diameter) by American Society for Testing Materials (A.S.T.M.)—Designation C 42–31. [b] Unit weight by nominal dimensions.

## Low-Casting Lifts

By careful control the dam was built up slowly over its full length by reasonably thin lifts and with long exposure periods to enable the "heat of hydration" to be dissipated before the lift was covered with another pour. The program involved 5-ft casting lifts with exposure periods of not less than five days, or equivalent to a one-foot height of concrete placed per day. It was believed that the additional time required for pouring thin lifts would not materially increase the cost.

Exceptions, however, were made to the general rule, particularly during the beginning of concrete placement on foundation rock or the resuming of concrete operations on old concrete; in such cases 2½-ft pours were standard practice. In general, 2½-ft pours were placed until approximately 60% of the foundation rock of each block was covered, except that on the steep foundation rock on the abutments practically the entire foundation was covered by using 2½-ft pours (Fig. 3). Further exceptions were made in some locations; for instance, in blocks 4 and 5 on the north abutment and blocks 23, 24, and 25 on the south abutment, marked changes in the regularity of the foundations formed abrupt slopes or humps. Where humps were found it was necessary to pour a rather large number of 2½-ft lifts before the top of the hump was covered. In order to permit the concrete to harden as long a period as possible and thus cool to about the temperature of the adjoining rock, exposure periods of four and five days were also adopted for 2½-ft pours. This program resulted in a successful treatment of these irregularities. Only one crack was observed as a result of the humps. This occurred in block 4 at a very sharp break in the foundation rock and where the concrete had to be relatively thin over the rock and exposed for almost a year.

From Table 4 it may be noted that, from the lowest area in the foundation in block 11 (El. 1,215.5) to the roadway elevation at the top of the dam (El.

1,237.6), a maximum difference in elevation of 322 ft is recorded and a total of sixty-nine pours was required giving an average of 4.66 ft. Sixty three of these were 5 ft high; two were $2\frac{1}{2}$-ft pours on the rock foundation; and four

(*a*) Beginning to Vibrate the Pile

(*b*) Vibrating Completed

Fig. 3.—Shallow Concrete Lifts ($2\frac{1}{2}$ Ft) on Foundation Rock

$2\frac{1}{2}$-ft pours were made on intermediate old concrete surfaces. In block 8 with an approximate height of 270 ft, a total of twenty six $2\frac{1}{2}$-ft pours was made, giving an average of 3.91 ft per pour. Similar data are given for all the blocks, with totals for the whole dam.

The small number of cracks is due in part to the care used in casting the concrete on the foundation in 2½-ft lifts. The considerable expense involved on account of practically doubling the cleanup cost seemed to be justifiable. Although the exposure time was permitted at two and a half days for the 2½-ft lifts, usually two 2½-ft pours were cast per week. Studies cited subsequently show the advantages of casting thin lifts with long exposure on foundation rock, thus reducing temperature rise.

TABLE 4.—SUMMARY REPORT—CONCRETE PLACEMENT, HIWASSEE DAM
(Number of Pours by 2½-ft and 5-ft Lifts)

| Block No. | Approximate height of block | No. of 5-ft pours | No. of 2½-ft pours | No. of intermediate 2½-ft pours | Total No. of pours | Total No. of 5-ft lifts on 5% slope upstream |
|---|---|---|---|---|---|---|
| 0 | 25 | 4 | 2 | .... | 6 | 5 |
| 1 | 35 | 4 | 6 | .... | 10 | 7 |
| 2 | 60 | 6 | 12 | .... | 18 | 12 |
| 3 | 82.5 | 16 | 1 | .... | 17 | 16.5 |
| 4 | 115 | 17 | 12 | .... | 29 | 23 |
| 5 | 155 | 21 | 20 | .... | 41 | 31 |
| 6 | 190 | 31 | 10 | 4 | 45 | 38 |
| 7 | 225 | 37 | 10 | 8 | 55 | 46 |
| 8 | 270 | 43 | 26 | .... | 69 | 56 |
| 9 | 295 | 51 | 16 | 4 | 71 | 61 |
| 10 | 295 | 55 | 16 | .... | 71 | 63 |
| 11 | 322 | 63 | 2 | 4 | 69 | 66 |
| 12 | 308 | 66 | 4 | .... | 70 | 68 |
| 13 | 310 | 60 | 6 | 2 | 68 | 64 |
| 14 | 305 | 60 | 8 | .... | 68 | 64 |
| 15 | 305 | 56 | 6 | 6 | 68 | 63 |
| 16 | 300 | 53 | 4 | 12 | 69 | 62 |
| 17 | 290 | 52 | 6 | 12.5 | 70.5 | 61 |
| 18 | 280 | 52 | 2 | 7 | 61 | 58 |
| 19 | 275 | 50 | 6 | 8 | 64 | 57 |
| 20 | 245 | 43 | 6 | 8 | 57 | 50 |
| 21 | 215 | 35 | 4 | 14 | 53 | 44 |
| 22 | 195 | 31 | 10 | 8 | 49 | 40 |
| 23 | 170 | 24 | 12 | 10 | 46 | 35 |
| 24 | 125 | 13 | 22 | 4 | 39 | 26 |
| 25 | 90 | 10 | 8 | 8 | 26 | 18 |
| 26 | 70 | 7 | 8 | 6 | 21 | 14 |
| 27 | 35 | 4 | 6 | .... | 10 | 7 |
| Totals | | 964 | 251 | 126 | 1,341 | .... |

## Low Casting Temperatures

It was known that the casting temperature of the concrete would be rather high, especially during the summer, unless something was done to reduce it artificially. Two expedients were adopted, (1) using refrigeration of the mixing water and (2) rinsing and cooling of the aggregate. The first was the more important. Water batches were taken from the insulated mixing plant surge tank that was supplied by an insulated pipe line from the refrigeration plant. An insulated overflow line returned the excess water to a tank at the refrigeration plant from which it was fed by gravity to sprinkler pipes discharging over the outside of the water cooler coils, thus securing second cooling contact. The water dripped off the cooler coils and ran into the pump sump ready for circulation. Approximately 50 gal per min of river water was cooled for concrete mixing from a usual temperature of about 80° F to as low as 35° F.

The construction of the refrigeration plant was begun in August, 1938, and was completed and put into service on September 2, but was shut down on

TABLE 5.—AVERAGE MONTHLY TEMPERATURES IN DEGREES FAHRENHEIT, CONCRETE AND CONCRETE MATERIALS

| Month | Number of Days Mixing Water Was: | | | Temperatures[a] | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Air | | Cement | | River water | Mixing Water | | | | | | Concrete Placing Temperature With: | | | | | |
| | Not treated | Heated | Cooled | | | | | | Not treated | | Heated | | Cooled | | Mixing water not treated | | Mixing water heated | | Mixing water cooled | |
| | | | | 1 | 2 | 1 | 2 | | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 | 1 | 2 |
| *1938:* | | | | | | | | | | | | | | | | | | | | |
| April | 7 | .... | .... | 70 | .... | 74 | .... | 60 | 63 | .... | .... | .... | .... | .... | 69 | .... | .... | .... | .... | .... |
| May | 21 | .... | .... | 76 | 65 | 80 | 85 | 65 | 66 | 66 | .... | .... | .... | .... | 71 | 69 | .... | .... | .... | .... |
| June | 22 | .... | .... | 80 | 77 | 98 | 95 | 72 | 72 | 72 | .... | .... | .... | .... | 76 | 74 | .... | .... | .... | .... |
| July | 20 | .... | .... | 79 | 72 | 102 | 106 | 69 | 72 | 73 | .... | .... | .... | .... | 79 | 80 | .... | .... | .... | .... |
| Aug. | 23 | .... | .... | 83 | 76 | 104 | 105 | 75 | 75 | 76 | .... | .... | .... | .... | 81 | 81 | .... | .... | .... | .... |
| Sept. | 10 | .... | 11 | 76 | 66 | 90 | 92 | 68 | 68 | 68 | .... | .... | 53 | 53 | 73 | 73 | .... | .... | 73 | 75 |
| Oct. | .... | .... | 21 | 67 | 58 | 92 | 93 | 59 | .... | .... | .... | .... | 42 | 43 | .... | .... | .... | .... | 63 | 61 |
| Nov. | 18 | 3 | .... | 54 | 48 | 76 | 78 | 52 | 50 | 53 | 112 | 106 | .... | .... | 55 | 57 | 46 | 49 | .... | .... |
| Dec. | 6 | 10 | .... | 44 | 35 | 68 | 71 | 43 | 46 | 44 | 85 | 89 | .... | .... | 49 | 50 | 50 | 51 | .... | .... |
| *1939:* | | | | | | | | | | | | | | | | | | | | |
| Jan. | 13 | 8 | .... | 46 | 40 | 72 | 75 | 42 | 45 | 45 | 80 | 87 | .... | .... | 51 | 47 | 50 | 51 | .... | .... |
| Feb. | 16 | 4 | .... | 53 | 46 | 75 | 80 | 43 | 47 | 49 | 115 | 107 | .... | .... | 52 | 53 | 53 | 53 | .... | .... |
| Mar. | 22 | .... | .... | 60 | 50 | 80 | 82 | 52 | 52 | 54 | .... | .... | .... | .... | 56 | 57 | .... | .... | .... | .... |
| Apr. | 20 | .... | .... | 64 | 54 | 74 | 75 | 56 | 58 | 58 | .... | .... | .... | .... | 59 | 60 | .... | .... | .... | .... |
| May | 10 | .... | 11 | 73 | 65 | 101 | 101 | 65 | 62 | 60 | .... | .... | 42 | 42 | 65 | 67 | .... | .... | 67 | 68 |
| June | 5 | .... | 17 | 83 | 74 | 107 | 107 | 75 | 76 | 76 | .... | .... | 44 | 45 | 81 | 81 | .... | .... | 74 | 74 |
| July | .... | .... | 19 | 85 | 75 | 104 | 105 | 77 | .... | .... | .... | .... | 43 | 46 | .... | .... | .... | .... | 75 | 75 |
| Aug. | .... | .... | 23 | 81 | 73 | 92 | 95 | 78 | .... | .... | .... | .... | 42 | 46 | .... | .... | .... | .... | 73 | 74 |
| Sept. | .... | .... | 20 | 79 | 69 | 96 | 105 | 77 | .... | .... | .... | .... | 41 | 43 | .... | .... | .... | .... | 72 | 70 |
| Oct. | .... | .... | 22 | 70 | 58 | 89 | 92 | 62 | .... | .... | .... | .... | 39 | 43 | .... | .... | .... | .... | 63 | 61 |
| Nov. | 13 | .... | 8 | 52 | 45 | 82 | 81 | 55 | 49 | 50 | .... | .... | 39 | 38 | 51 | 48 | .... | .... | 51 | 50 |
| Dec. | 8 | 10 | .... | 49 | 39 | 79 | 80 | 43 | 44 | 45 | 84 | 89 | .... | .... | 50 | 50 | 53 | 53 | .... | .... |
| *1940:* | | | | | | | | | | | | | | | | | | | | |
| Jan. | .... | 22 | .... | .... | 26 | .... | 46 | 36 | .... | .... | .... | 134 | .... | .... | .... | .... | .... | 54 | .... | .... |
| Total | 234 | 57 | 152 | | | | | | | | | | | | | | | | | |

[a] Divided into columns marked 1 and 2 and referring to: Shift 1, 6:00 a.m.–3:00 p.m.; shift 2, 3:00 p.m.–12 midnight

October 23, 1938, due to cold weather. In Table 5 are given various temperature data. During winter operations, the mixing water was heated by steam, its temperature being raised from approximately 40° F to 130° F. During a cold spell, the temperatures of the water ran as high as 180° F.

## Washing, Rinsing, and Cooling the Aggregate

To control the grading, rinsing screens were installed to wash all aggregate except sand. In this process the aggregate was thoroughly wetted and subsequently cooled by a blast of cold air while it was dropping from the rinsing screens to the conveyer belt. This had the effect of cooling it approximately 3°. The effective reduction in concrete placing temperature by refrigerating the mixing water was about 6°. For a 5-ft lift during an exposure of five days the effective reduction in maximum temperatures due to refrigeration of the mixing water was calculated at about 3°, and due to the washing and cooling of the aggregate about 1°. The placing temperature of the concrete in the summer months averaged about 70° F, and for the winter months every effort was made to secure a temperature between 45° F and 50° F. In view of the small reductions in maximum temperature due to refrigerating the mixing water, it seems that better results might be obtained by installing an effective system of cooling for the aggregate and other ingredients.

## Artificially Cooling the Concrete in Place

In general, resort was made to artificial cooling of the concrete in place (1) to reduce temperature cracking that might occur on high exposed contraction joint faces; (2) where the exposure period on the usual 5-ft pours was reduced on account of the exigency of the schedule; (3) to equalize the temperatures between the concrete on the foundation and the adjoining rock as quickly as possible; and (4) to reduce the temperature of concrete in closed recesses to bring it as quickly as possible to that of the adjoining concrete. In block 19 (Fig. 4) about 15 gal per min of river water flowing through each of three loops composed of one-inch iron pipes were used to artificially cool a 5-ft lift of concrete in place. Better results were obtained when the water was of low temperature. The difference in temperature between inflowing and outflowing water ranged from 4° to as high as 10° F. The flow as a rule was regulated daily and the cooling at each level was continued for fifteen days or more.

In Table 6 are given applicable data on artificial cooling. The cofferdam bulkheads in blocks 17 and 18 were poured with slightly thicker lifts than usual so cooling pipes were laid and water circulated in three lifts. This served to (1) expedite the pouring of concrete in those two temporary spillway blocks, (2) eliminate cracking because these bulkheads were designed as integral parts of the dam structure, and (3) offset the constant danger from destructive floods. The concrete in the assembly recesses around the ring seal gates was heavily reinforced and artificially cooled. Thus the maximum time was saved in the pouring schedule by equalizing the temperatures of the green with the older concrete; and high temperatures were prevented in parts of the gate castings when other parts were relatively cool, thus preventing warping in the castings and the gate. The first two 5-ft lifts of closure blocks 10–12–14 (spillway

section) were artificially cooled in order to eliminate the necessity of making these pours in $2\frac{1}{2}$-ft lifts. As a large structural steel gate was utilized in the closure, a program of pouring in $2\frac{1}{2}$-ft lifts would have been awkward. The concrete in block 11 was cooled to prevent cracks due to irregularities in the

FIG. 4.—GALLERY FORMS AND COOLING PIPES IN BLOCK 19

TABLE 6.—SUMMARY OF CONCRETE LIFTS ARTIFICIALLY COOLED

| Block No. | Lifts Cooled | | Vertical Location, by Numbered Lifts[a] | | Remarks |
|---|---|---|---|---|---|
| | No. | Thickness, in ft | From | To | |
| 3 | 12 | 5 | 13 | 2 | Pouring 2 lifts per week |
| 9 | 5 | $2\frac{1}{2}$[b] | 51 | 49 | Ring seal gate recess only |
| 10 | 2 | 5 | 54 | 53 | In lieu of four $2\frac{1}{2}$-ft lifts |
| 11 | 5 | 5 | 64 | 60 | Due to foundation conditions |
| 11 | 6 | $2\frac{1}{2}$[b] | 51 | 49 | Ring seal gate recess only |
| 12 | 2 | 5 | 54 | 53 | In lieu of four $2\frac{1}{2}$-ft lifts |
| 13 | 6 | $2\frac{1}{2}$[b] | 51 | 49 | Ring seal gate recess only |
| 14 | 2 | 5 | 54 | 53 | In lieu of four $2\frac{1}{2}$-ft lifts |
| 15 | 6 | $2\frac{1}{2}$[b] | 51 | 49 | Ring seal gate recess only |
| 17 | 3 | 7 | 54 | 50 | Bulkhead only |
| 17 | 3 | 5 | 53 | 51 | Adjacent to bulkhead |
| 18 | 3 | 7 | 54 | 50 | Bulkhead only |
| 18 | 3 | 5 | 53 | 51 | Adjacent to bulkhead |
| 19 | 17 | 5 | 46 | 30 | High bulkhead wall |

[a] Cooling pipes were laid on top of one lift and covered by the next. Lift numbers refer to covering lifts. Numbering of 5-ft lifts started with No. 1 just below roadway level and extended downward to No. 68 at maximum depth. [b] Approximately.

foundation and in the shape of the block itself. Nearly all of the concrete in block 3 was cooled so that a more rapid pouring schedule might be adopted; this block contained the transfer track and, therefore, was not poured until many of the blocks in the dam and the adjoining blocks had almost reached the

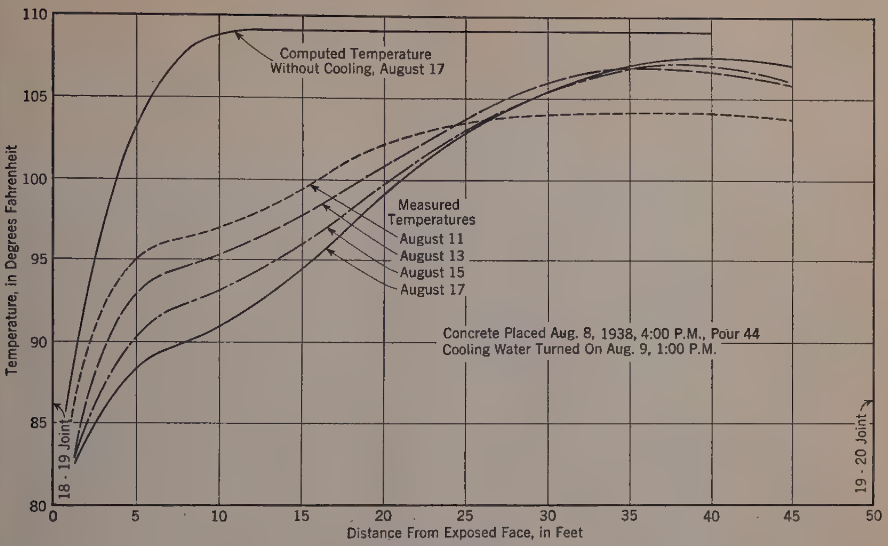


FIG. 5.—TEMPERATURE DISTRIBUTION PARALLEL TO AXIS OF DAM; BLOCK 19, BOTTOM OF POUR 44, EL. 1,320.7

ultimate top elevation of the dam. Therefore, it was poured, utilizing 5-ft lifts with two pours weekly and artificial cooling.

The curves in Fig. 5 show the temperature distribution parallel to the axis of the dam, at the bottom of pour 44 in block 19, both with and without artificial cooling by the circulation of water. It is apparent that the artificial cooling of the concrete after placing was justified.

## CLEANING HORIZONTAL CONSTRUCTION JOINTS BETWEEN LIFTS

Great care was exercised in preparing old concrete surfaces and foundation rock before pouring in order to secure a good bond, and thus prevent cracking and percolation along the joint with the resulting high uplift pressure. For the first three months a wet sandblast only was used, but on account of the high cost, due partly to the large number of 2½-ft lifts, other methods were attempted.

The method which was finally adopted consisted of an initial "green cut," with a jet of air and water, of the entire surface from 6 to 8 hr after the concrete had been poured in the summer, and from 8 to 20 hr in the winter; and then a final cleanup over the entire area with air and water, which also included a wet sandblast cut of that area from the upstream face of the dam to a line approximately 5 ft below the 8-in. formed drains, with special sandblasting treatments of large areas of laitance. The cutting with a high velocity jet of air and water was performed with an air pressure of about 50 lb and a water pressure of about 100 lb. The final cleanup usually came just prior to resuming concrete placement in the block. It was essential that the initial and final cleanup be followed by a thorough washing and a clean-off by air to remove loose material and water. The cleanup on the job as a whole was excellent.

Usually, the placing of concrete began at the upstream face and progressed downstream on a rising 5% grade. At times, therefore, the cleanup could be performed free from interference with the concrete placing. Before concrete was placed on the rock foundation or on old concrete the surface was usually wetted and about $\frac{3}{4}$ in. of cement-sand mortar was well broomed into the surface. The wetting of the old concrete was abandoned for a time, but during the summer season the surfaces dried out very rapidly so the grout tended to roll during the brooming process. Mass concrete was thoroughly compacted in the forms by means of internal vibrators handled by two men and operated by 3-phase electric power. Although the vibrators were rated at 60 cycles, the frequency was changed during the progress of the work to 90 cycles, or about 5,400 vibrations per min. Great care was exercised in order that the new concrete was thoroughly compacted from the top of the lift to the surface of the old concrete. Any surplus cement-sand mortar was vibrated into the concrete and thus a good bond secured. Under normal operating conditions five of these large vibrators were used and compacted about 125 yd per hr. The concrete, when deposited from a $7\frac{1}{2}$-cu-yd bucket (by air valve release), formed a pile of very harsh and coarse looking concrete, $3\frac{1}{2}$ or 4 ft high (Fig. 3), which would be vibrated down to a thickness of $1\frac{1}{2}$ or 2 ft. Care was taken that the concrete be not dumped from an elevation higher than 15 ft above the casting grade.

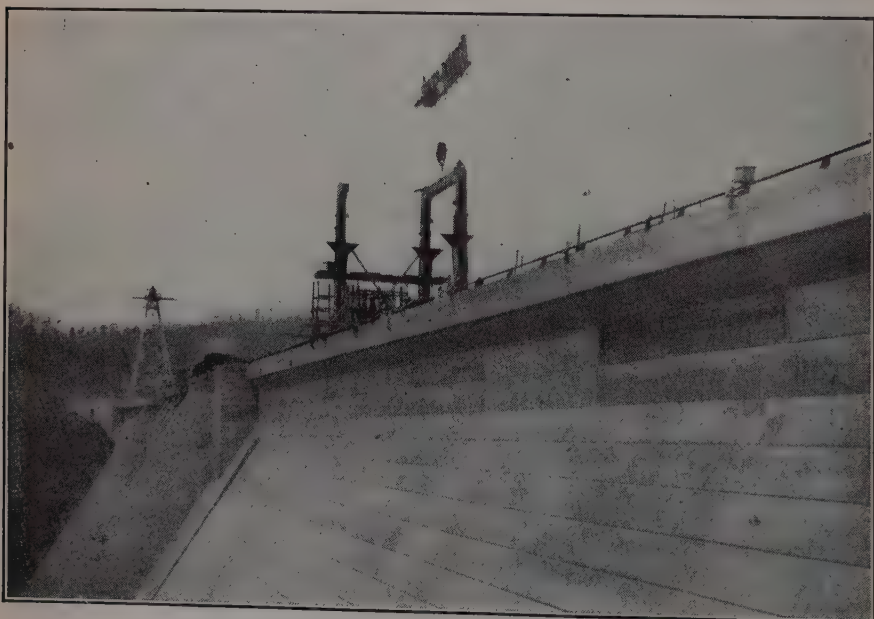

FIG. 6.—DOWNSTREAM FACE, SOUTH ABUTMENT, SHOWING HORIZONTAL CHAMFERED JOINTS, AND CRANE ASSEMBLY ON TOP

To improve the appearance of the dam, to break the usual monotony of a bare concrete face to such great height, and to eliminate any conspicuous appearance of cracks at the joints between pours, a system of horizontal and

vertical chamfered joints at lift and block lines was inaugurated. This gives the appearance of a dam with blocks 50 ft wide by 5 ft high (see Fig. 6). The horizontal chamfered joints were not placed on the downstream surface of the spillway section, but they were placed on the upstream face, as well as on the adjacent abutments, above the drawdown level (El. 1,415). The vertical chamfered joints on the abutments were $3\frac{3}{4}$ in. wide and $1\frac{1}{4}$ in. deep as compared with 2 in. by 0.75 in. for the spillway. The horizontal chamfered fillets were $3\frac{3}{4}$ in. by $1\frac{1}{4}$ in. On blocks where a large number of $2\frac{1}{2}$-ft pours were scheduled, 5-ft pours were cast for a distance of approximately 20 or 30 ft from the downstream face and then the surface was sloped down to make a $2\frac{1}{2}$-ft pour through the balance of the block. The distances of 20 and 30 ft from the downstream faces were staggered so as to eliminate the possibility of a vertical crack forming parallel to the axis of the dam. As far as is known cracking was obviated by this procedure.

To prevent the percolation of water along the contraction joints from the upstream face, vertical 6-in. drains were constructed, with water stops of stainless steel sheets extending through the drains and 12 in. on both sides into the concrete.[2] These extended from the rock foundation to the top of the dam. Additional strips of stainless steel sheets 8 in. wide and 3 ft long were welded to the stainless steel water stops on each side of the contraction joint, and extended 4 in. into the top of each lift, thus bonding the two pours and preventing the seepage of water along the horizontal pour joints with resultant cracking.

Except for the closure blocks 10, 12, and 14 and for some slight adjustments in blocks 17 and 18 due to their use as a temporary spillway and to complications during the construction of the penstock, concrete was poured on a 5% grade (Fig. 2) sloping upward from the upstream face toward the downstream face. No joints or breaks in these lifts were permitted except when they were

TABLE 7.—INTERRUPTIONS IN CONCRETE POURING

| Date | Shift | Duration | Block No. | Cause |
|---|---|---|---|---|
| 8-4-38 | Night | 5 hr | 23 | Excessive rain flooded out freshly poured concrete causing abandonment of form |
| 12-20-38 | Night | $34\frac{1}{2}$ hr | 7 | Conveyer pulley shaft broke at head house |
| 7-12-39 | Night | $11\frac{1}{2}$ hr | 18 | Changing endless cable on cableway |
| 3-23-39 | Night | $6\frac{1}{2}$ hr | 18 | Block too large to complete in regular shift |
| 3-28-39 | Night | 8 days[a] | 8 | A $2\frac{1}{2}$-ft pour left incomplete, cableway trouble |
| 12-14-38 | Day | $20\frac{1}{2}$ hr | 14 | Head pulley and shaft broken on conveyer belt to mixing plant |

[a] Did not complete half lift.

unavoidable on account of breakdowns in the plant or on account of a flood, as in the case of a closure block. Only six of these inside joints were made in the total number of 1,341 pours in the dam. Under no consideration were these

joints permitted closer than 20 ft from either face of the dam; and where the exposure period was in excess of approximately 6 hr, the surfaces were cleaned up and covered with grout the same as a new pour. It is certain that no longitudinal crack appeared in the dam as a result of these joints. Irregularities in pouring are listed in Table 7.

## Steel Reinforcement to Prevent Cracking

Careful inspection of the 1,827.6 lin ft of galleries in the dam has disclosed very few cracks. This is due to several contributing circumstances, one of which is the placing of heavy steel reinforcement, from 1-in. to $1\frac{1}{4}$-in. square bars, in a horizontal position 5 in. above the roof of the gallery (Fig. 7). No

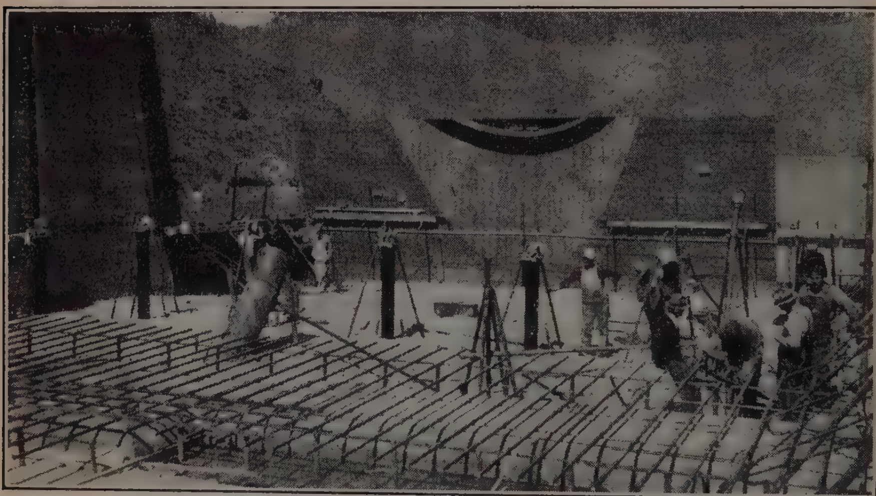

Fig. 7.—Typical Reinforcement over Roof of Inspection Gallery

doubt other contributing conditions included the use of low-heat cement; the low cement content of mass concrete that was placed all around the inspection galleries regardless of its close proximity to the upstream face of the dam; the long periods of exposure between pours; the pre-cooling of the mixing water in warm weather; and the use of flat roof with corner fillets on a 12-in. radius rather than a full arch.

Steel reinforcement was also used in the recesses provided for the placement of the ring seal gates in order to prevent cracking in the comparatively thin walls of concrete between the gates and the contraction joints and to bind the concrete firmly to the regulating conduits and the ring seal gate castings. A few cracks were found in the bulkhead faces of blocks 4, 6, 7, 15, 19, and 20, and several of these cracks extended through more than one lift. The first crack was discovered (September 23, 1938) on the 19–20 contraction joint face of block 20. This was caused, no doubt, by a rather large height differential between the two blocks (about 30 ft), and by a rather sudden drop in temperature from a low of 60° at night to a low of approximately 40°. Two cracks

were found on the bulkhead face of block 19, which also were caused by the excessive difference (maximum 130 ft) between the top of blocks 18 and 19.

It was anticipated that the high face in block 19 would be subject to some cracking. Therefore, sixteen 5-ft lifts were cooled by pipes for a total height of 80 ft. Two cracks occurred in this bulkhead face, the largest being 0.1 in. It was interesting to note that this crack terminated after it penetrated the cooled portion of the block.

The largest crack occurred in block 15 on the 14–15 contraction joint about November 28, 1938. This crack was influenced, no doubt, by the large ring seal gate recess in the center of block 15, but the 60-ft difference in height between the tops of blocks 14 and 15, coupled with a sudden drop in the minimum

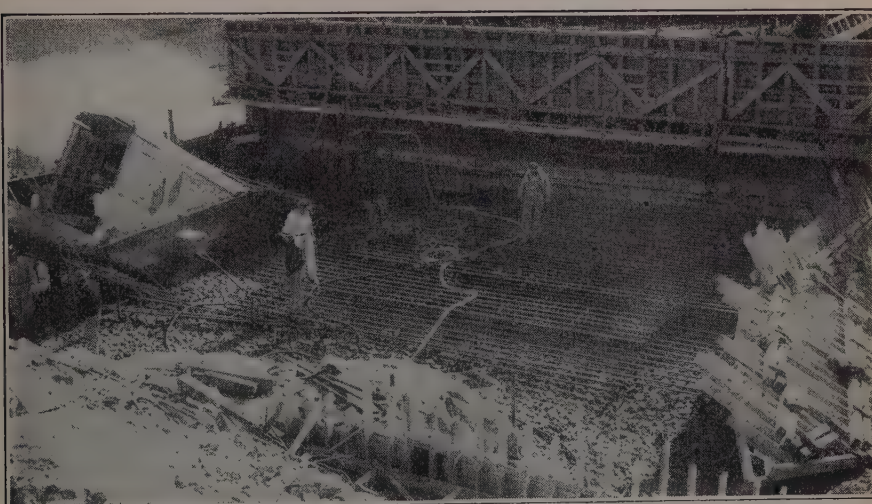

Fig. 8.—Mat of Steel Reinforcement Placed in Block 4 to Prevent Upward Extension of Crack

air temperatures, probably caused it. This crack reached a maximum opening of $\frac{1}{8}$ in. and extended to El. 1,298 and also across the top of the lift parallel to the axis and into the ring seal gate recess. As it was more nearly of structural significance than any other crack, it was carefully explored and full data obtained as to the location, depth, and width inside the concrete. An attempt was made to stop this crack from projecting into the pours that would be made above El. 1,298. Another crack was discovered in block 4, which was caused by the existence of a hump in the rock under the concrete, a considerable height differential, and a long exposure of the block.

The crack in block 15 occurred on the side of the bulkhead wall, where it was possible to observe it from time to time. It was decided to place heavy mats of reinforcing steel on the top of the lift and across the block in an attempt to keep these cracks from extending upward. In general, it may be stated that the mat in block 15 was composed of $1\frac{1}{4}$-in. square bars placed 9 in. center to center, both vertically and horizontally, in cross sectional areas parallel to the

axis, and $\frac{3}{4}$-in. spacer bars 9 in. vertically and 2 ft 6 in. horizontally. A mat for block 4 was similar (Fig. 8). These mats proved very effective, as it was possible to observe that the crack in block 15 did not extend beyond the top of the block at El. 1,298 where the steel was placed.

In addition, a decision was made to treat cracks wider than 0.03 in. by covering them with tar impregnated canvas to prevent the entrance of mortar and foreign matter. Horizontal holes were then drilled into the concrete of the bulkhead wall to intersect the crack about a foot from the surface, and pipes were laid for grouting these cracks at a later date.

## Diagonal Keyways on Bulkhead Joints

During the construction of Norris Dam a number of cracks on the bulkhead walls extended to the downstream face of the dam following the vertical keyways. To overcome this tendency, diagonal keyways were designed for Hiwassee Dam on the bulkhead walls following the direction of the maximum

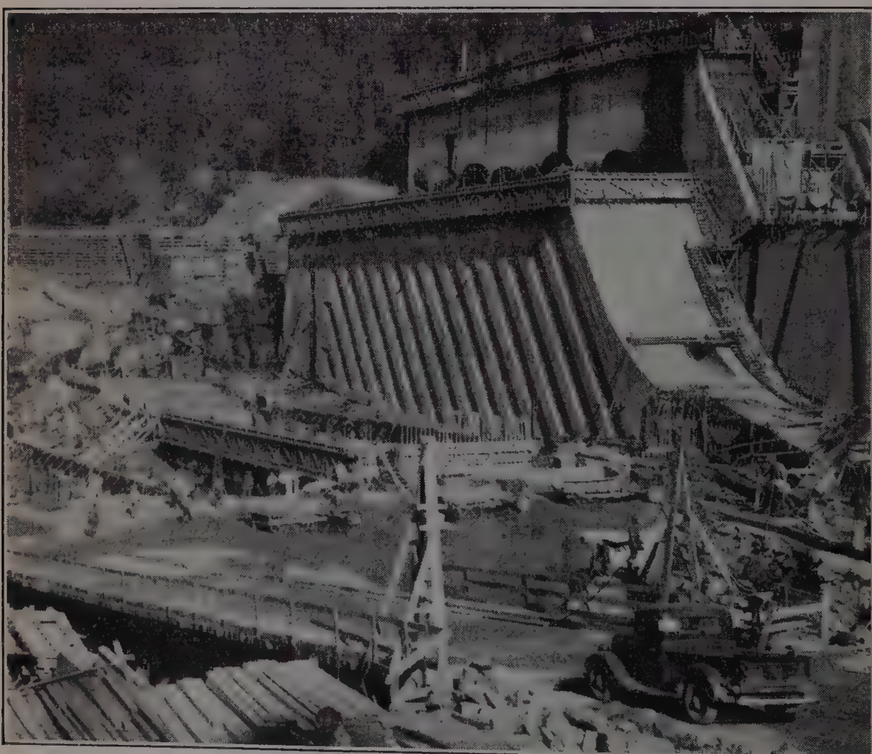

Fig. 9.—Diagonal Keys on Bulkhead Wall of Block 15 (Spillway Section)

stresses (practically parallel to the downstream face, see Fig. 9). As a rule, the metal forms used for these keyways were tacked securely to the face of the bulkhead panels to keep them attached during the stripping operations.

The keys have a trapezoidal cross section, 6 in. deep and 4 ft wide at the bottom with 12 in. for each chamfered slope.[3] They were constructed of 14-gage and 16-gage steel plate, and on an inclination of $7\frac{5}{32}$ in. horizontally to 12 in. vertically. It is difficult to form any positive conclusion as to the effectiveness of these keyways to prevent cracking on the downsteam face because there were so few cracks at Hiwassee Dam and none of the cracks on the bulkhead faces extended to the downstream face. On the contraction joints, however, it was noticed that fine cracks would follow the keyways, but in almost all instances cracks that extended through several lifts crossed the keyways.

## Curing and Winter Protection

A program of curing was adopted for Hiwassee Dam at the outset. It consisted of wetting the top of the blocks and the sides for a period of twenty-one days. This did not include the bulkhead walls nor was it done as a rule when the air temperature was 40° F or lower. For a time, sprinklers were used on top of the blocks, and then hand sprinkling. Iron pipes with small holes about 6 in. center to center were fastened to the panel forms to spray both upstream and downstream faces of the dam.

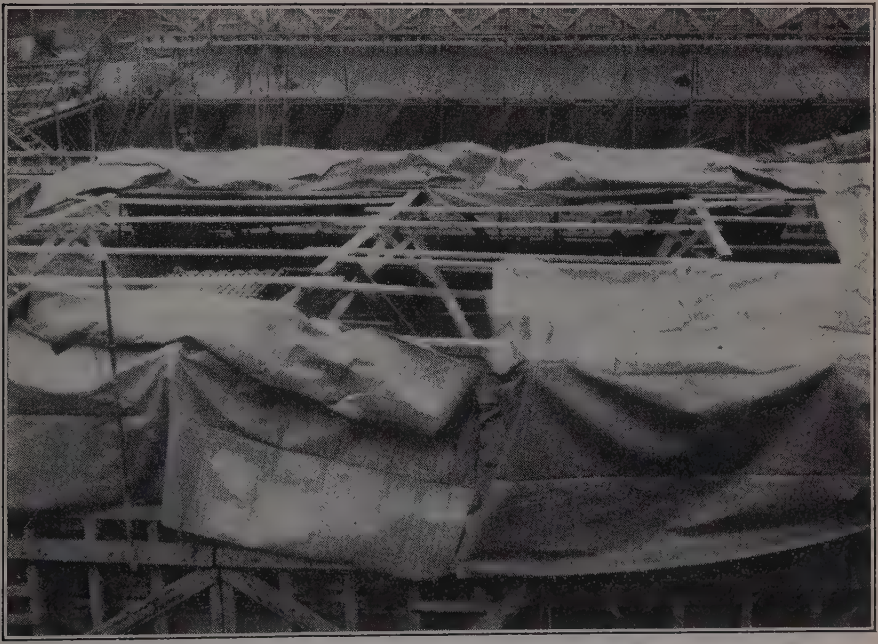

Fig. 10.—To Protect Concrete in Freezing Weather a Tarpaulin Cover was Supported on Horses, with Steam Coils Underneath

Efficient curing by sprinkling with water for long periods undoubtedly produced a continuous gain in strength; and this prevented major cracking as well as fine checks or hair cracks. These hair cracks, when subjected to sudden and

[3] "Design of Hiwassee Dam—Engineering Details," by Cecil E. Pearce, *Civil Engineering*, July, 1940, p. 435.

wide variations in temperature, might develop into real cracks. It is believed, therefore, that an adequate program of curing did materially assist in the crack prevention program.

Due to the slower hardening process of low-heat cement, it was necessary to install rather elaborate equipment for winter protection not only for the aggregate but also for the mixing and placing of concrete. The gates in the aggregate reclaiming tunnel were surrounded with steam pipes and the tunnel itself was heated by steam. The rinsing screens were protected by wooden housing built around the entire equipment and heated by electricity. The Mixer Building was heated, also the aggregate in the bins, by means of steam coils. The mixing water tank was provided with steam coils. After the concrete had been mixed and delivered to the forms, it was protected from freezing by coverings of heavy tarpaulins or canvas (Fig. 10) and by the use of large steam coils placed sufficiently above the concrete to prevent its damage due to drying out. Much cold weather was experienced, but no serious damage was done to the dam or spillway structure.

## Temperature Studies

An attempt has been made to determine by what amount the maximum temperature in the dam has been reduced by the various expedients adopted. From temperature observations it was ascertained that the maximum temperature rise in the dam was 40°. Calculations based on observations at

TABLE 8.—Estimated Effects on Maximum Temperature Rise Due to Various Adjustments

| No.: | Normal Schedule: | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | Lifts, in ft | 10.0 | | | 10.0 | | | 5.0 | | | 5.0 | | | 2.5 | | |
| 2 | Days per lift | 3.0 | | | 5.0 | | | 3.0 | | | 5.0 | | | 3.5 | | |
| 3 | Average, in ft per day | 3.33 | | | 2.00 | | | 1.67 | | | 1.00 | | | 0.71 | | |
| | Temperature Reduction in Degrees F for Various Types of Cement (S = Standard; M = Modified; L = Low Heat) | | | | | | | | | | | | | | | |
| | | S | M | L | S | M | L | S | M | L | S | M | L | S | M | L |
| | Reduction: | | | | | | | | | | | | | | | |
| 4 | Normal schedule | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | Normal schedule, but L cement | 15 | 7 | .. | 13 | 6 | .. | 12 | 5 | .. | 9 | 4 | .. | 7 | 3 | .. |
| 6 | Normal schedule, but 5-ft lifts every five days | 13 | 11 | 8 | 9 | 7 | 5 | 7 | 5 | 4 | .. | .. | .. | −5[a] | −4[a] | −3[a] |
| 7 | Normal schedule, but 2.5-ft lifts every 3.5 days | 18 | 15 | 11 | 14 | 11 | 8 | 12 | 10 | 8 | 5 | 4 | 3 | .. | .. | .. |
| | Combinations (with L Cement): | | | | | | | | | | | | | | | |
| 8 | 5-ft lifts every five days | 22 | 15 | 8 | 18 | 11 | 5 | 16 | 9 | 4 | 9 | 4 | .. | 4 | 0 | −3[a] |
| 9 | 2.5-ft lifts every 3.5 days | 26 | 18 | 11 | 21 | 14 | 8 | 19 | 12 | 8 | 12 | 7 | 3 | 7 | 3 | .. |

[a] Minus sign indicates an increase in temperature.

Hiwassee Dam, Norris Dam, and elsewhere, seem to indicate that the use of low-heat cement, low cement content (0.8 bbl per cu yd), low casting lifts ($2\frac{1}{2}$ ft) with three and a half days exposure, artificial cooling of mixing water, aggregate,

and concrete in place have reduced the maximum temperature approximately 31° below that which could have been expected had "Type B" (modified cement) been used, with a cement content of one barrel per cubic yard, and 5-ft lifts and five-day exposure periods. Of course, this would apply only to those parts of the dam constructed in the $2\frac{1}{2}$-ft lifts and using artificial cooling of the mass. Table 8 shows estimated changes in the maximum temperature rise due to the placing rate and type of cement used; it gives temperatures for concrete, poured at Hiwassee Dam, containing 0.8 bbl of cement per cubic yard, based on preliminary estimates of heat generation for low-heat cement. The cement actually used generated more heat than was estimated. In addition, Figs. 5 and 11 show temperature variations, both recorded and computed.

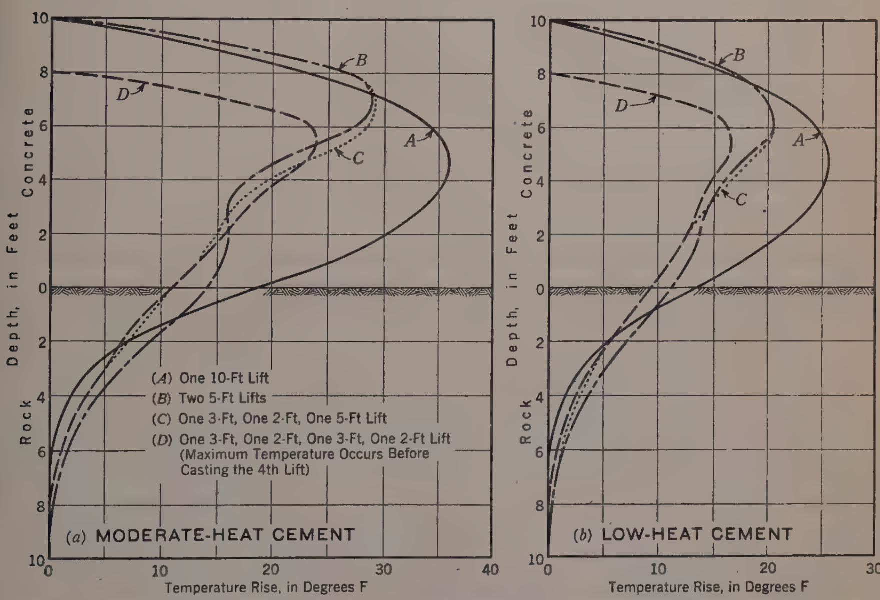


Fig. 11.—Temperature Computations. Curves Plotted for Time of Maximum Temperature; Cement Content, 0.8 Bbl per Cu Yd; Average Placing Rate, 1.0 Ft per Day

During construction a rather complete recording of concrete temperatures, strains, joint openings, measurements by electric detectors, deflections, and uplift pressures was conducted. A unique testing program was undertaken, in cooperation with the University of California, at Berkeley, Calif., to aid in interpreting the data from strain measurements, and to determine the stresses by laboratory methods rather than by mathematical analyses. The entire scientific program was directed by the TVA materials engineering organization, except the experiments at the university. This scientific program was not necessary at Hiwassee Dam in order to prosecute the crack prevention program. That is, when the results were finally interpreted by the experiments and cal-

culations, it was too late to apply them; but they may be of service to other engineers and on other large projects.

## Cost of Program

A comparison of the cost of the program of crack prevention at Hiwassee Dam, with a program that would have used a predicted cement content of one barrel per cubic yard and without all of the crack prevention features, has been made. In reviewing the cost, the additional life of the structure, which has no doubt been increased because of the relative absence of cracks, should also be considered and credited, especially when the crack prevention program has actually resulted in a saving.

*Additional Costs.*—To conduct the scientific program cost $29,000, which includes all instruments, equipment, special shop work, labor, salaries, and also the University of California cooperative work. The use of low-heat cement has increased the cost, by adding 7½ cents per bbl for 766,375 bbl, or a total of $58,115. In placing the two hundred and fifty-one 2½-ft pours, an additional area of 643,691 sq ft of cleanup was required which, at 6 cents per sq ft, increased the cost by $38,621. The cost incurred by the erection of the refrigerating plant and all of its accessories, including the cost of operation, involved an expenditure of $41,820. Only a small proportion of the total cost of $48,677 for the equipment, materials, and cost of erection, as well as the operation, of the washing, rinsing, and drying plant for the aggregate can be directly chargeable to the cooling of the aggregate; it has been determined that $278, or 3% of the installation cost, and $2,781, or 50% of the cost of compressed air used, or a total of $3,059 is chargeable to the cost of this feature. The cost of the materials, supplies, including pipe, fittings, and installation, as well as the operation of the system for artificially cooling some of the concrete blocks, has been determined as $11,864. The additional cost of the steel reinforcement under the floor and above the ceiling of the 1,828 lin ft of galleries in the dam amounted to 238,210 lb at 4.64 cents per lb, or $11,033; and the cost of installing the two mats of steel reinforcement to prevent the two cracks in blocks 4 and 15 from extending upward involved the placement of 52,201 lb of steel at 4.64 cents per lb, or $3,814, making the total of these two items $14,857. The total cost of the curing was $43,500 of which $10,875 was charged to the crack prevention program, and the total cost of winter protection was $31,600, of which $3,200 was charged to the crack prevention program, making a total of $14,075 charged to these items.

*Reduced Cement Content.*—In the placement of 596,431 cu yd of mass concrete (as differentiated from face concrete), a saving of 114,499 bbl of cement was effected over the amount of cement that would have been required had the predicted one barrel of cement been used for each cubic yard of mass concrete. When a check was made by deducting 0.20 bbl of cement per cubic yard for the 596,451 yd of mass concrete, a saving of 119,290 bbl was obtained. In calculating the saving in cement, however, the lower number (114,499 bbl) was used; at $2.14 per bbl, which is the cost at the mixing plant, a total saving of $245,028 is indicated.

The cost of the crack prevention program may be summarized as follows: Additional cost of—

| | |
|---|---|
| Scientific program | \$ 29,000 |
| Use of low-heat cement | 58,115 |
| Thin casting lifts | 38,621 |
| Low casting temperatures | 41,820 |
| Cooling the aggregate | 3,059 |
| Artificially cooling the concrete in place | 11,064 |
| Use of steel reinforcement | 14,867 |
| Curing and winter protection | 14,075 |
| Total additional cost of crack prevention (794,439 cu yd at 26½ cents) | \$210,621 |
| Reduction in cost for saving in cement (114,499 bbl at \$2.14) | 245,028 |
| Surplus or saving | \$ 34,407 |

## Acknowledgments

During construction I. L. Tyler, M. Am. Soc. C. E., Douglas H. McHenry, and W. R. Waugh, Assoc. M. Am. Soc. C. E., have had charge of the Materials Engineering Section in the order named under the immediate direction of the writer. These men supervised the work in the concrete laboratory, the technical control of the aggregate manufacture, and the concrete mixing and placing; conducted the scientific program; and directed much of the work on the crack prevention program. The results achieved are largely due to their efforts.

Hiwassee Dam was designed and built by the TVA under the general direction of: A. E. Morgan as chief engineer and Carl A. Bock as assistant chief engineer until May 1, 1938, and since that date, Theodore B. Parker, chief engineer; B. W. Steele until November 20, 1936, as chief design engineer; Barton M. Jones as acting chief design engineer from November 20, 1936, to January 16, 1939; H. A. Hageman since that date as chief design engineer; Theodore B. Parker to July 1, 1938, and A. L. Pauls, since that date, as chief construction engineer; C. E. Blee, project engineer at the dam; and the writer as construction engineer. All the foregoing are members of the Society. Lex Phifer was acting construction superintendent to January 1, 1938; from that date to March 8, 1940, J. E. Walters was construction superintendent, and since the last date E. C. McClenagan was acting construction superintendent.

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PAPERS

# FORMULAS FOR THE TRANSPORTATION OF BED LOAD

By H. A. Einstein,[1] Assoc. M. Am. Soc. C. E.

Synopsis

A method for the representation of bed-load data is given in this paper. The method is based on the conception that bed-load transportation is the movement of bed particles, as governed by the laws of probability. By means of this method an equation is obtained, which describes a great number of experiments in channels with uniform beds. A group of experiments conducted on sand mixtures provides material for describing another application of the method.

## Introduction

In the past the problem of bed-load transportation has been studied mostly by empirical methods. More recently, there has been a tendency to base transportation formulas on the new theories of turbulence.[2,3] It is the writer's belief that an approach to the problem of transportation can be made by a combination of the empirical and rational methods and that the results can be expressed by dimensionless plots.

Before proceeding with the development of these formulas, it is necessary to discuss briefly two important considerations: (1) The difficulty or impossibility of defining, accurately, the so-called "critical" values; and (2) the possibility of correlating bed-load movement with local fluctuations in water velocity along the bed.

(1) Attempts have been made in the past to derive an expression for the "initial movement" that is governed by certain definable "critical" conditions to be used as the first step toward the solution of the transportation problem. In interpreting the results of many experiments on bed-load movement, and

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **July 15, 1941.**

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[2] "Anwendung der Aehnlichkeitsmechanik und der Turbulenzforschung auf die Geschiebebewegung," by A. Shields, *Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau*, Heft 26, Berlin, 1936.

[3] "An Analysis of Sediment Transportation in the Light of Fluid Turbulence," by Hunter Rouse, Assoc. M. Am. Soc. C. E., SCS, Sedimentation Div., SCS-TP-25, Washington, D. C., July, 1939 (mimeographed).

in comparing them with those obtained by other experimenters, the writer has concluded that a distinct condition for the beginning of transportation does not seem to exist. It is just as impossible to determine the limit of initial movement as to determine the maximum possible flood of a river. Just as the engineer is able to predict the probable maximum flood of a river to be expected within a given range of years, however, so is he able to define the hydraulic conditions in a stream that will produce any given small rate of movement, which might be called the limit. This value can be chosen without any restriction. It is difficult to believe that the hydraulic conditions that will produce such movement could have any special meaning in the problem of transportation as a whole. Therefore, the writer will not use the conception of critical tractive force, or any other critical value, when the term "critical" pertains to the flow where transportation begins.

(2) In general, transportation of bed load has been described as follows:[4] A particle of the bed moves when the pushing force or lifting force of the water overcomes the weight of the particle. This push or "lift" is expressed in terms of the average flow. The usual conception is that transportation begins when the velocity increases enough to overcome the weight, and that, with further increasing velocity of the water, the rate of transportation will also increase, following a certain law that is found empirically. To prove that this conception is misleading, assume that the force acting on a particle could be described by means of the average flow alone. If the velocity of the water is increased gradually to a point at which the first particle would just be moving, the force acting on all the other particles of the same kind and size would move those too. Therefore, in a uniform bed where all particles have the same size and shape, all would start moving together; they would be unable to settle again because at all points the water velocity is just sufficient to start movement. If this were true, there could be no law governing the rate of transportation, but only a critical velocity. At all undercritical velocities, there would be no transportation, whereas, at all supercritical velocities, the rate of transportation would be limited only by the number of particles available. Therefore, it cannot be presumed that the rate of bed-load transportation is a function of the average flow. Instead it is proposed to express it in terms of the fluctuations of the water velocity near the bed.

Results of previous studies[5] describing the movement of a bed-load particle by means of statistical methods are to be used in an attempt to coordinate the rate of transportation with the fluctuations of the water velocity near the bed. The results of these studies can be summarized as follows:

(*a*) These flume studies dealt with the movement of rather coarse particles along a bed consisting of the same kind of grains. Being coarser than 1 in. these particles always remained near the bed, rolling, sliding and, sometimes, in saltation according to the normal description of bed-load movement. It was found that the moving bed load and the bed on which it was moving formed

[4] "The Force Required to Move Particles on a Stream Bed," by William W. Rubey, *Professional Paper No. 189-E*, U. S. Geological Survey, Washington, D. C., 1938.

[5] "Der Geschiebetrieb als Wahrscheinlichkeitsproblem," von H. A. Einstein, *Mitteilung der Versuchsanstalt für Wasserbau an der Eidgenössische technische Hochschule in Zürich*, Verlag Rascher & Co., Zürich, 1937.

a unit, inasmuch as there was a steady and intensive exchange of particles between the two. Thus it is concluded that all the particles of the bed, down to a certain depth, take equal part in the movement, alternatively moving and returning into the bed.

(*b*) Bed-load movement is to be considered as the motion of bed particles in quick steps with comparatively long intermediate periods of rest. Thus bed-load movement is a slow downstream motion of a certain top-layer of the bed.

(*c*) The average step of a certain particle seems always to be the same even if the hydraulic conditions or the composition of the bed changes; and

(*d*) Different rates of transportation are produced by a change in average time between two steps and by a change in the thickness of the moving layer.

These concepts permit the development of a formula in general terms. The rate of transportation will be described by means of this average "step."

## Notation

The letter symbols in this paper are defined where they first appear and are assembled for convenience of reference in Appendix I.

## Derivations

This paper will treat only the bed-load movement of uniform sediment and mixtures acting like uniform sediment. In both cases it is possible to describe the sediment by a representative diameter $D$ and its density $\rho_s$. The expression "acting like uniform material" means that both bed material and moving material have the same composition and, therefore, the same representative diameters. It is possible in this case to describe transportation at a certain point of the bed by one symbol; namely, the rate of transportation $q_s$. Two special cases of transportation, both of which have been observed and described in engineering literature, may be excluded from treatment in this paper: (1) Bed material moving in suspension; and (2) bed-load movements, in which the composition of the bed is essentially different from the composition of the transported material.

A bed-load formula is an equation linking the rate of bed-load transportation with properties of the grain and of the flow causing the movement. A formula of this kind can be derived by expressing in an equation the fact that all particles passing the unit width of a section as bed load are just on the way to perform one of these steps of the constant length $L = \lambda_0 D$. Fig. 1 shows the cross section and the rectangular area with the length $L$ and with unit width, where all particles start the steps that together form the rate of transportation $q_s$. Eq. 1 expresses the condition that the total volume passing the unit width of the section per second ($q_s$ divided by the specific gravity of the particles, both under water)

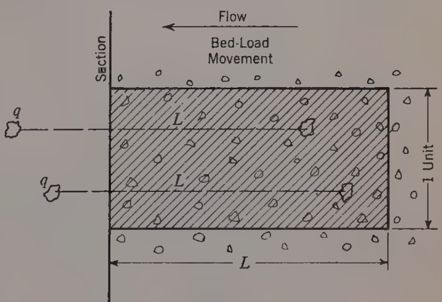



Fig. 1

equals the total volume of all the particles starting a step during a second in the aforementioned rectangular area. This volume is obtained by multiplying the number of particles in the surface of the area by the probability that a particle in the bed surface will start moving during a given instant and with the volume of a given particle. Eq. 1 follows:

$$\frac{q_s}{(\rho_s - \rho_f)\,g} = \frac{L}{A_1\,D^2}\,p_s\,A_2\,D^3 = \frac{A_2}{A_1}\,\lambda_0\,p_s\,D^2 \ldots\ldots\ldots\ldots (1)$$

in which: $q_s$ = the rate of transportation, in weight (under water), per unit of width, per second; $\rho_s$ and $\rho_f$ = density of particle and fluid, respectively; $g$ = acceleration due to gravity; $D$ = representative diameter of the particles; $A_1$ and $A_2$ = dimensionless ratios such that $A_1\,D^2$ = the area that the grain covers in the bed and $A_2\,D^3$ = the volume of the particle; $p_s$ = the probability that a particle will start moving in any given second; and $\lambda_0$ = the dimensionless measure for the length of the single step.[5] It must be kept in mind, however, that $\lambda_0$ may or may not be a constant; that is, it has not been proved to be constant.

If $A$ and $D$ are transposed to the left side of Eq. 1, that side will include all terms pertaining to the grain, whereas the right side $\lambda_0\,p_s$ is still an unknown function of the flow—that is:

$$\frac{q_s}{(\rho_s - \rho_f)\,g\,D^2}\,\frac{A_1}{A_2} = \lambda_0\,p_s \ldots\ldots\ldots\ldots (2)$$

In this equation $A_1$, $A_2$, and $\lambda_0$ are dimensionless, but $p_s$ has the dimension $\sec^{-1}$. In order to make the right side of Eq. 2 dimensionless, $p_s$ must be multiplied by a given time. The most reasonable time to use is the average time, $t$, required for the water to remove one particle from the bed. If $p = t\,p_s$, then $p$ is dimensionless and gives the number of steps that start from any given place during the time it takes to remove one particle. The maximum value of $p$ is 1, and indicates that at all times and at all points particle by particle starts to move. The minimum value of $p$ is zero; therefore, $p$ expresses the probability that a step is about to begin at a given place. These steps start everywhere on the bed. Therefore, $p$ can be interpreted as the probable part of the bed area in which steps are starting. A step is started only at a point where the hydraulic lift of the water is able to overcome the weight of the particle. Therefore, $p$ expresses the probability that the hydraulic lift on any particle along the bed is about to overcome the weight of the particle.

Unfortunately there is no method of expressing or measuring the time $t$ required for the lifting force to pick up a particle. It is assumed to be proportional to some other characteristic time of the particle in the water. The time that the particle requires to settle in water, a distance equal to its own diameter $D$, is chosen for this characteristic. The reason for choosing this time was the fact that it is the only expression with the dimension of a time, which is representative for the behavior of the particle in the liquid without including any characteristics of the flow. This time can be expressed as

$$\frac{D}{v_f} = \frac{1}{F}\sqrt{\frac{D\,\rho_f}{g\,(\rho_s - \rho_f)}} \ldots\ldots\ldots\ldots (3)$$

in which: $v_f$ = the velocity of a particle settling in water; and $F$ = a parameter for settling velocity. In Eq. 3, $F = 0.816$ for particles greater than 1 mm, settling in water of normal temperature. Fig. 2 shows the values of $F$ for smaller grain sizes. Characteristics of the materials in Fig. 2 are: Kinematic viscosity $\nu = \frac{\mu}{\rho_f}$ equals, for water, 0.012, and, for air, 0.16, $cm^2$ per sec; and specific densities, $\frac{\rho_s - \rho_f}{\rho_f}$, are as follows—

| Material | Specific density |
|---|---|
| Barite in water | 3.22 |
| Gravel in water | 1.65 |
| Coal in water | 0.25 |
| Gravel in air | 2,210.0 |

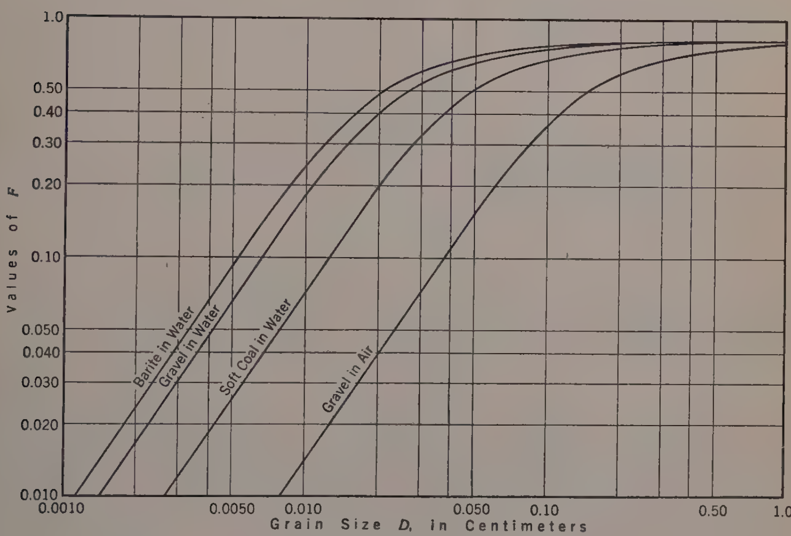



FIG. 2.—PARAMETER $F$ FOR DETERMINING THE SETTLING VELOCITY OF VARIOUS MATERIALS

In Fig. 2 the following equation for the settling velocity derived by William W. Rubey[6] has been used for the determination of $F$:

$$v_f = \sqrt{\frac{2}{3} g \frac{\rho_s - \rho_f}{\rho_f} D + \frac{36\,\mu^2}{\rho_f^2\,D^2}} - \frac{6\,\mu}{\rho_f\,D} = F \sqrt{D\,g \frac{\rho_s - \rho_f}{\rho_f}} \quad \text{.....(4)}$$

In this formula all terms are measured in centimeter-gram-second units. Hence, $F$ will be

$$F = \sqrt{\frac{2}{3} + \frac{36\,\mu^2}{g\,D^3\,\rho_f\,(\rho_s - \rho_f)}} - \sqrt{\frac{36\,\mu^2}{g\,D^3\,\rho_f\,(\rho_s - \rho_f)}} \quad \text{........(5)}$$

[6] "Settling Velocities of Gravel, Sand, and Silt," by William W. Rubey, *American Journal of Science*, Vol. 25, No. 148, April, 1933.

The time $t$ required to remove a particle from its place in the bed then will be

$$t = \frac{A_3}{F}\sqrt{\frac{D\,\rho_f}{g\,(\rho_s - \rho_f)}} = \frac{p}{p_s} \qquad (6)$$

in which $A_3$ is still an unknown constant. Eq. 1 can now be changed to the form

$$p = \frac{A_1\,A_3}{\lambda_0\,A_2}\left[\frac{1}{F}\,\frac{q_s}{(\rho_s - \rho_f)\,g}\sqrt{\frac{\rho_f}{\rho_s - \rho_f}}\,\frac{1}{g^{0.5}\,D^{1.5}}\right] \qquad (7)$$

In an attempt to express $p$ as the probability of the local hydraulic lift[4] to overcome the weight of the particle, $p$ refers to the part of the bed in which locally (at a certain moment) the lifting force is greater than the weight of the particle. It can be stated that $p$ refers to the part of the bed in which the ratio of the local lift to the average lift is greater than the ratio of the weight of the particle to the average lift. In mathematical terms this is

$$p = f\left(\frac{\text{Weight of the particle}}{\text{Average lift of the particle}}\right) \qquad (8)$$

in which $f$ is an unknown function. The weight of the particle under water is $A_2\,D^3\,(\rho_s - \rho_f)\,g$, and the average lift is

$$\text{Lift} = A_4\,D^2\,v^2\,\rho_f \qquad (9)$$

$v$ being a local velocity at some still unknown distance from the bed. An approximate value for $v$ is:

$$v = 11.6\sqrt{\frac{\tau}{\rho_f}} \qquad (10)$$

Eq. 10 may need to be corrected in the future. It defines the velocity[3] at the edge of the laminar boundary layer if the wall is smooth, or the velocity at the distance $D$ if the wall is rough, in which $D$ is a measure of average roughness. The shearing stress $\tau$ along the wall is

$$\tau = S\,R\,\rho_f\,g \qquad (11)$$

in which $S$ is the slope and $R$ is the hydraulic radius; therefore

$$v = 11.6\,\sqrt{S\,R\,g} \qquad (12)$$

Eq. 8 can now be written

$$p = f\left[\frac{A_2\,D^3\,(\rho_s - \rho_f)\,g}{(A_4\,D^2\,\rho_f)\,(135\,S\,R\,g)}\right] = f\left[\frac{A_2}{135\,A_4}\times\frac{(\rho_s - \rho_f)^D}{\rho_f\,S\,R}\right] \qquad (13)$$

By assuming that Eq. 10 gives the correct value for the velocity, Eqs. 7 and 13 can be combined and a new transportation formula formed:

$$A\left\{\frac{1}{F}\left[\frac{q_s}{(\rho_s - \rho_f)\,g}\right]\sqrt{\frac{\rho_f}{\rho_s - \rho_f}}\,\frac{1}{g^{0.5}\,D^{1.5}}\right\} = f\left[B\left(\frac{\rho_s - \rho_f}{\rho_f}\,\frac{D}{S\,R}\right)\right] = p. \qquad (14)$$

in which

$$A = \frac{A_1 A_3}{\lambda_0 A_2} \qquad (15a)$$

and

$$B = \frac{A_2}{135 A_4} \qquad (15b)$$

are constants which, however, may vary with different shapes of the particles. Whether $\lambda_0$ and $A_4$ really are constant under all conditions must be determined later. Introducing

$$\phi = \frac{1}{F}\frac{q_s}{(\rho_s - \rho_f)\,g}\sqrt{\frac{\rho_f}{\rho_s - \rho_f}}\frac{1}{g^{0.5}\,D^{1.5}} \qquad (16a)$$

and

$$\psi = \frac{\rho_s - \rho_f}{\rho_f}\frac{D}{S\,R} \qquad (16b)$$

Eq. 14 can be written in the short form:

$$A\,\phi = f(B\,\psi) = p \qquad (17)$$

The function $f$ as well as the two constants $A$ and $B$ must be determined empirically. Data from a great number of measurements using various materials have been analyzed, and values of $\psi$ and $\phi$ computed. A semilogarithmic plot of these values is shown in Fig. 3($a$). The grain sizes range from 0.315 to 28.65 mm in diameter, the water depth from 18 to 1,100 mm, and the specific gravity of the particles from 1.25 to 4.22. All these experiments are performed in flumes with uniform sediment. The experiments marked "Zürich" are described briefly in a paper by E. Meyer-Peter[7] whereas the remaining experiments are taken from the generally known paper by G. K. Gilbert.[8] It seems that all these points follow, reasonably, a single curve. It should be noted that from these data not a single experiment has been omitted even when the measurement appeared questionable. It might be mentioned also that the hydraulic radius $R$ is computed by a method[9] that eliminates the effect of side-wall friction and gives results comparable to a channel of infinite width (see Appendix II).

In Fig. 3($a$) all the points with values of $\phi$ less than 0.4 seem to follow the straight line, curve (1), the equation of which is

$$0.465\,\phi = e^{-0.391\,\psi} \qquad (18)$$

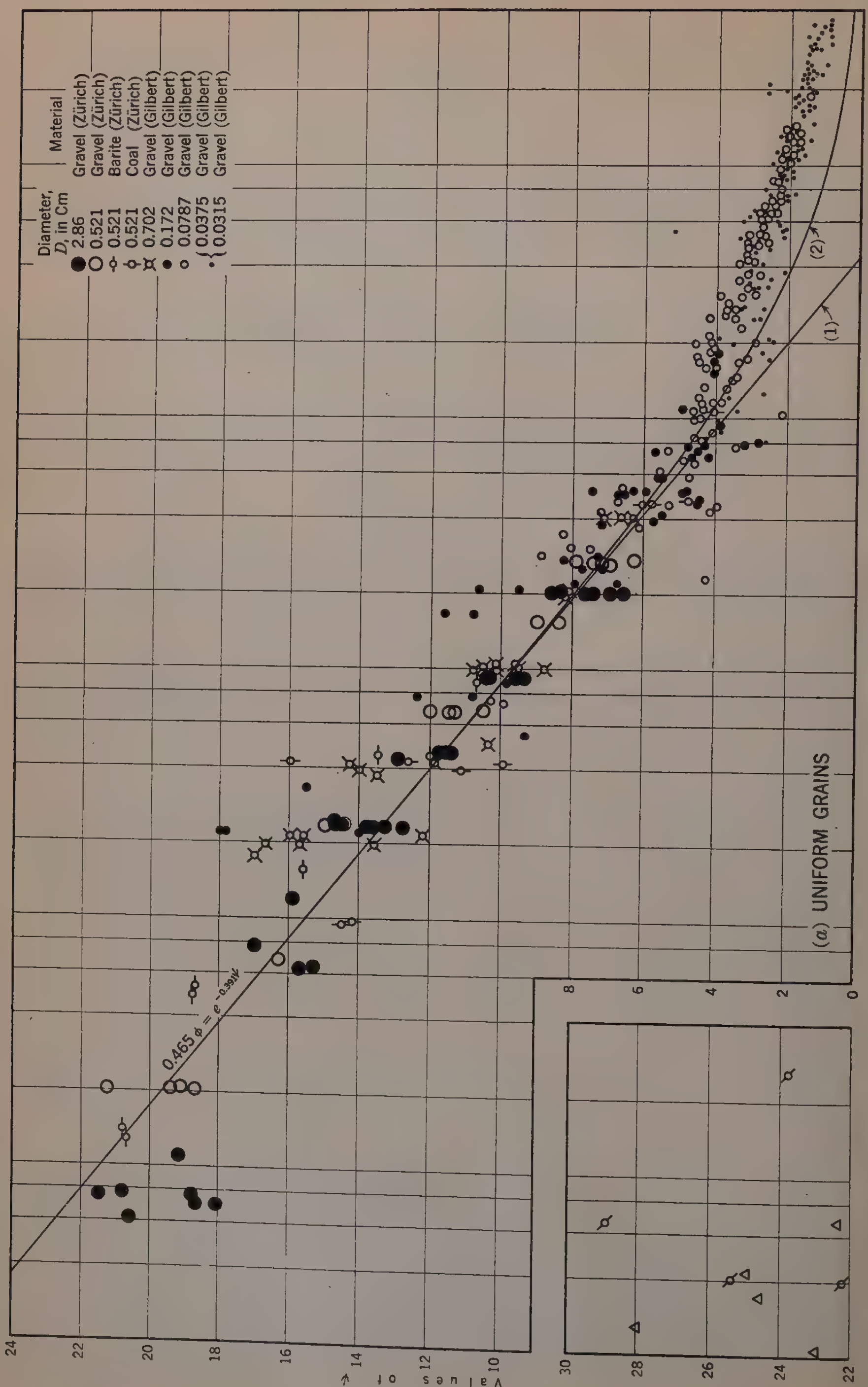
If Eq. 18 is assumed to represent the law of transportation:

$$\left.\begin{aligned} A &= 0.465 \\ B &= 0.391 \\ f(x) &= e^{-x} \end{aligned}\right\} \qquad (19)$$

[7] "Neuere Versuchsresultate über den Geschiebetrieb," by E. Meyer-Peter, H. Favre, and A. Einstein, *Schweizerische Bauzeitung*, Vol. 103, No. 13, March, 1934.

[8] "The Transportation of Debris by Running Water," by Grove Karl Gilbert, *Professional Paper No. 86*, U. S. Geological Survey, Washington, D. C., 1914.

[9] "Der hydraulische oder Profil-Radius," by H. A. Einstein, *Schweizerische Bauzeitung*, Vol. 103, No. 8, February 24, 1934; see also Appendix II.



Diameter, D, in Cm | Material
2.86 Gravel (Zürich)
0.521 Gravel (Zürich)
0.521 Barite (Zürich)
0.521 Coal (Zürich)
0.702 Gravel (Gilbert)
0.172 Gravel (Gilbert)
0.0787 Gravel (Gilbert)
0.0375 Gravel (Gilbert)
0.0315 Gravel (Gilbert)
0.465 φ = e^-0.391ψ
(1)
(2)
(a) UNIFORM GRAINS
Values of ψ

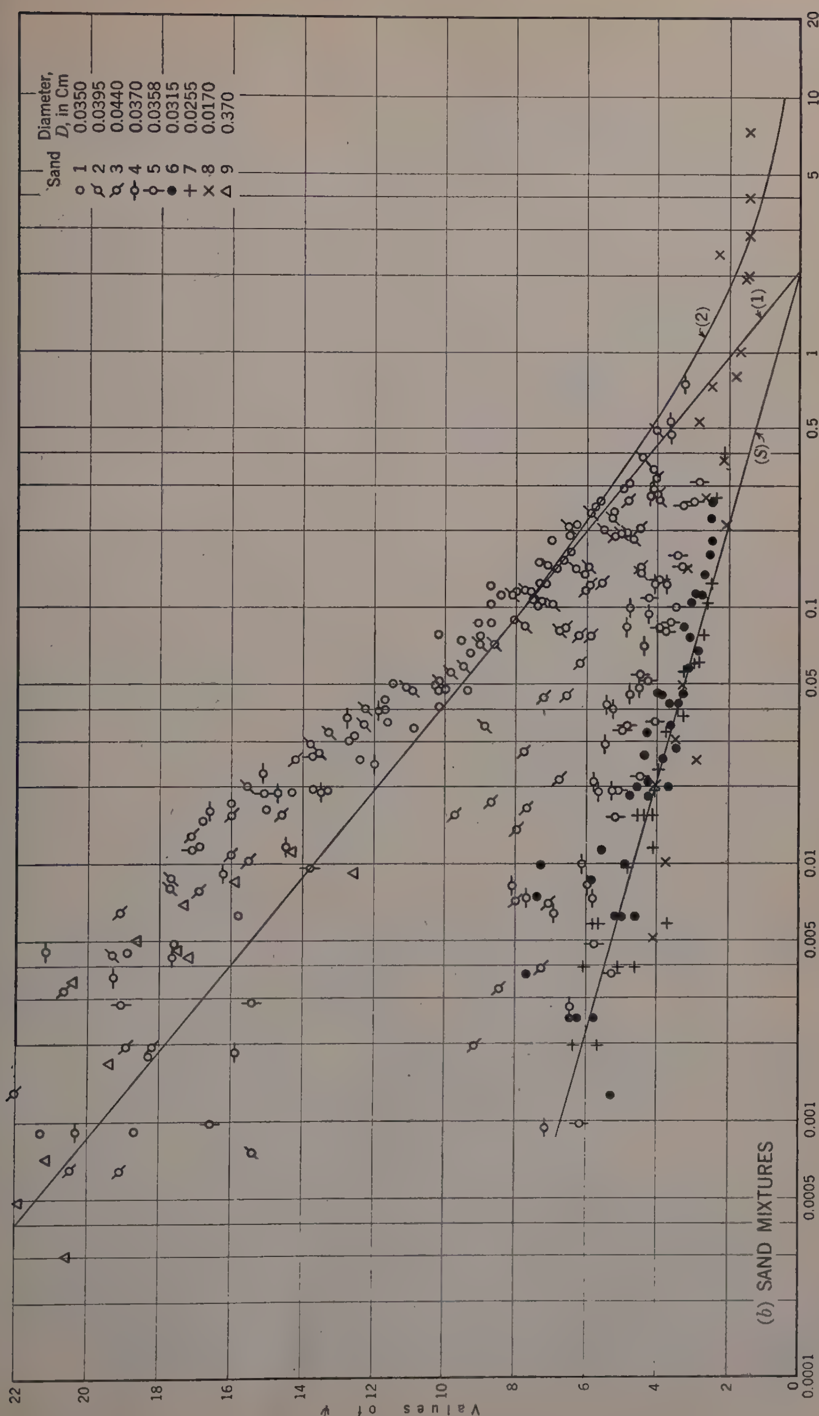



Fig. 3.—Bed-Load Experiments Showing the Relation Between $\phi$ and $\psi$

it remains only to explain why the points $\phi > 0.4$ seem to be too high. If Eq. 18 is accepted as a general law, the value $\phi > 2.15$ would not be possible because $p$ cannot exceed 1. Therefore, values of $\phi > 2.15$ are possible only if $A$ becomes smaller ($A$ consists of the constants $A_1$, $A_2$, $A_3$, and $\lambda_0$).

The constants $A_1$, $A_2$, and $A_3$ are not likely to change with increasing values of $p$, but $\lambda_0$ does. The distance $\lambda_0$ has been found to be constant for small values of $p$—that is, when the hydraulic lift seldom exceeds the weight. As $p$ increases, it more often happens that, in the very spot where the step would have ended, there exists a local lift strong enough to keep the particle from settling. The oftener this happens the more $\lambda_0$ seems to increase on the average. The symbol $p$ expresses the probability that the lift exceeds the weight of the particle for every point on the bed. Therefore, only $(1 - p)$ particles of the unit will be able to settle after a step $\lambda_0$. The other $p$ particles will start for another $\lambda_0$, and out of these $(1 - p)\, p$ will settle after the second $\lambda_0$, and so on. The average distance traversed by the unit, therefore, is:

$$\lambda = \sum_{m=0}^{\infty} (1 - p)\, p^{m-1}\, m\, \lambda_0 = \frac{\lambda_0}{1 - p} \qquad (20)$$

in which $m$ = a whole positive number. Introducing $\lambda$ instead of $\lambda_0$ yields curve (2) instead of curve (1). This new curve follows the plotted points more closely than curve (1).

The constant $A_4$ for the lift in Eq. 9 is introduced without further discussion. The question arises whether the deviation of the points from curve (2) could be due to a change in the constant $A_4$. Constant $A_4$ could change only with the Reynolds number of the flow around the particle, or with the value of $\frac{D}{\delta}$, the ratio of the grain size to the thickness of the laminar layer. Eq. 10 gives the local velocity of the water:

$$v = 11.6 \sqrt{\frac{\tau_0}{\rho_f}} = 11.6 \sqrt{S\, R\, g} \qquad (21)$$

The Reynolds number of the local flow is

$$\mathbf{R} = \frac{D\, v}{\nu} = \frac{11.6\, D \sqrt{S\, R\, g}}{\nu} \qquad (22)$$

and the thickness of the laminar layer is

$$\delta = \nu \frac{D\, v}{\tau_0/\rho_f} = \frac{11.6\, \nu}{\sqrt{S\, R\, g}} \qquad (23)$$

or

$$\frac{D}{\delta} = \frac{D \sqrt{S\, R\, g}}{11.6\, \nu} = \frac{\mathbf{R}_D}{134} \qquad (24)$$

As $\frac{D}{\delta}$ differs from $\mathbf{R}_D$ only by a constant factor, it is sufficient to study the influence of only one of them. The deviation of the measured points from curve (2) plotted against $\frac{D}{\delta}$ failed to disclose any satisfactory relationship. Each

grain size appears to follow a separate curve; therefore, it appears much more probable that $A_4$ is a constant, but that the exponential law for $p$ does not extend down to $\psi = 0$. Another explanation for the deviation of the points may be that part of the grains have been transported in suspension. In this case those experiments would be outside the field of application of Eq. 17. More experiments, however, will be required in this range to determine the exact shape of the curve. It may be emphasized that in most American rivers the bed load is largely transported under conditions pertaining to this part of the curve, and, therefore, further study in this direction is most urgent.

The title of this paper was chosen specifically to avoid the impression that any attempt was being made to discover "the law of bed-load transportation," because it is the writer's belief that such universal law does not exist in a simple mathematical form. Just as it is necessary to distinguish between friction in smooth and rough pipes or channels, so it is necessary to distinguish between different kinds of transportation. Nevertheless, the distinction between friction along rough, wavy, and smooth walls was only possible on the discovery of the general method of plotting the friction factor against Reynolds' number. An attempt is made in this paper to determine a corresponding method of presenting transportation data by introducing the quantities $\psi$ and $\phi$, both of which are derived by pure speculation, using only generally known facts.

As an example, the method is used to discuss the results of experiments with sand mixtures, conducted at the U. S. Waterways Experiment Station, at Vicksburg, Miss.[10] Fig. 3(*b*) gives the results of these experiments as a $\phi$-$\psi$ graph. The method described in Appendix II has been used to determine the effective hydraulic radius. Curves (1) and (2) are transferred from Fig. 3(*a*), and all the various sand mixtures have been assigned different symbols.

The first problem was to determine the effective diameter of the mixtures—that is, the value of $D$ that would represent the mixture in the formulas. Experience gained in previous studies has convinced the writer that the most usable value for this effective diameter is the grain size of which 35% to 45% of the material is finer. This value is readily obtained from the cumulative size-frequency curve of the mixture. The 40% value was used for Fig. 3(*b*), although it is realized that the use of a 35% value would tend to bring the high points closer to curves (1) and (2).

The distribution of the points in Fig. 3(*b*) is very interesting to note. At first glance one observes that the points for sands 1, 2, and 9 distinctly follow curves (1) and (2). Sands 3, 4, and 5 follow the curves in the upper part only. Sands 6, 7, and 8 fall below the curves at all points, but a distinct grouping of points along a line curve ($S$) is noticed.

The two curves, (2) and ($S$), seem to represent limits of maximum and minimum transportation for a given value of $\psi$. In searching for an explanation of this the three following questions naturally arise: (1) Is there any relationship between the position of the points and friction loss? (2) Is there any relationship between the position of the points and the condition of the bed?

[10] "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River," *Paper No. 17*, U. S. Waterways Experiment Station, Vicksburg, Miss., January, 1935.

(3) Would a similar distribution be possible also in experiments with uniform material, or is it characteristic only of mixtures?

With regard to question (1), it was found that Manning's $n$, without exception, increased suddenly when the points leave curves (1) and (2). This means that the bed becomes rougher than the original material as soon as the rate of transportation decreases below that shown by curves (1) and (2). The reverse is also true—that is, the rate of transportation will decrease as soon as the roughness of the bed increases.

The reason for this increased roughness is of interest. As a rule, riffles begin to form precisely at the place where the points depart from curve (1). If riffles are the reason for the increase in Manning's $n$, this value must always increase when riffles are formed. Sand 1 does not show this increase and sand 2 only very slightly—although these sands develop general riffles like all the other mixtures. Therefore, the riffles are not the reason for the increased roughness, but merely happen to develop simultaneously. This answers question (2) in the negative.

Question (3) suggests that perhaps some kind of sorting of the grains is the reason for the deviation from curve (1) and for the increase of roughness at the same time. It is the writer's belief that this is true, but unfortunately it is not subject to direct proof. This sorting would be caused by the lack of material in the upper end of the flume. If the sand is fed in at a smaller rate than the stream is able to transport it, the bed starts to build some kind of a protective layer of coarse grains on the surface and buries all the finer grains beneath. The average grain size in this coarse layer is much greater than the average grain size in the bed, and scour is either reduced or completely prevented. For this reason, it is impossible, during an experiment, to detect a lack of feeding merely by watching the position of the bed. Curve (1) gives the results obtained when the highest quantity of sand is fed in that can be transported without deposition, and curve ($S$) gives the smallest amount that will be transported without scour on the protecting layer. If $D$ is the effective diameter of the original bed material, it is reasonable to believe that these two limits coincide for uniform material and that curve ($S$) falls more and more below curve (1) as the material decreases in uniformity.

These are merely some suggested methods of studying bed-load transportation. It would be very easy to determine, by experiment, the validity of such reasoning. If the interpretations are correct, it should be possible to determine all points between the two limiting curves by merely changing the rate of sand feed. It would also be very instructive to conduct a similar group of experiments in the opposite sequence—that is, by beginning with high discharges and progressively decreasing the discharge and load. If the explanation is correct, one will probably not return to the same curve obtained with increasing flow. This would also answer question (3). This $\phi$-$\psi$ method is offered as a new procedure for studying bed-load problems. It may be possible to refine the method by introducing corrections for the velocity $v$, and various constants; but as a whole it seems to be satisfactory in its present form.

## Summary

In concluding it may be stated that the treatment of transportation problems by means of statistical methods, made possible by the use of large-scale experiments, led to the proposed method of representation:

Two dimensionless functions $\psi$ and $\phi$ have been developed theoretically, $\psi$ as the ratio of the forces acting on the particle, and $\phi$ including the rate of transportation and the size and settling velocity of the particle. The interrelation between these two functions expresses the law of transportation, and at the same time expresses the statistical distribution $p$ of the velocity of the liquid close to the laminar boundary layer. The transportation law is derived by use of a great many experiments with uniform sediment, and is then used in discussing published results of experiments with sand mixtures.

## Acknowledgments

The writer is greatly indebted to Prof. E. Meyer-Peter of Zürich, Switzerland, for having permitted the use of valuable data, even though the results of the original work have not as yet been published in full.

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# APPENDIX I

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## Notation

The following notation conforms essentially with American Standard Symbols for Hydraulics compiled by a Committee of the American Standards Association[11] with Society representation, and approved by the Association in 1929:

$A$ = a dimensionless constant, defined in Eq. 15$a$:

- $A_1 D^2$ = the area that the grain covers in the bed; and
- $A_2 D^3$ = the volume of the grain;
- $A_3$ = a dimensionless unknown constant in Eq. 6;
- $A_4$ = a dimensionless unknown constant in Eq. 13, the expression for the hydraulic lift of a particle;
- $A_b$ = the area with reference to the bed;
- $A_m$ = the area with reference to part $m$ of a cross section;
- $A_w$ = the area with reference to the wall;

$B$ = a dimensionless constant, defined in Eq. 15$b$;

$b$ = surface width of a stream;

$D$ = diameter; the representative diameter of a particle;

$d$ = depth; depth from the water surface to a particle;

$e$ = base of Napierian logarithms;

$F$ = a factor defined by Eq. 5;

$f$ = function of; see also $f_m$ in Eq. 25:

- $f_b$ = function for friction along the bed;
- $f_w$ = function for friction along the wall;

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[11] ASA–Z10b–1929.

$g$ = acceleration due to gravity;
$L$ = length of the steps taken successively by particles in a river bed $= \lambda_0 D$, the average of all the steps taken by a single particle;
$m$ = a whole number;
$N$ = a number of units;
$n$ = Manning's roughness coefficient:
  $n_b$ = roughness of the river bed;
  $n_m$ = roughness of part $m$ of the perimeter;
  $n_w$ = roughness of the side-wall;
$P$ = wetted perimeter:
  $P_b$ = wetted perimeter along the bed;
  $P_m$ = wetted perimeter of part $m$ of the circumference;
  $P_w$ = part of the wetted perimeter along the wall;
$p$ = probability that, at a given point in the bed, the lifting force required to overcome the weight of the particle has been generated:
  $p_s$ = the probability of a particle starting movement at a given second;
$q$ = rate of flow or discharge per unit width:
  $q_b$ = discharge pertaining to the unit width of bed;
  $q_s$ = the rate of transportation, in weight (under water), per unit of width, per second;
$R$ = hydraulic radius:
  $R_b$ = hydraulic radius pertaining to the bed;
  $R_m$ = hydraulic radius pertaining to the part $m$ of the cross section;
  $R_w$ = hydraulic radius pertaining to the wall;
$\mathbf{R_D}$ = Reynolds' number of the grain $D$;
$S$ = hydraulic slope;
$t$ = time required for the water to build up sufficient force to pick up a particle;
$V$ = average velocity in the cross section of a stream;
$v$ = filament velocity at a given point in the cross section and at an unknown distance from the bed:
  $v_f$ = settling velocity;
$\delta$ = thickness of the laminar layer (see Eq. 23);
$\lambda$ = average distance traveled by a unit of bed load:
  $\lambda_0$ = the dimensionless measure of a single step taken by a particle;
$\mu$ = absolute viscosity of a fluid;
$\nu$ = kinematic viscosity $= \frac{\mu}{\rho_f}$;
$\rho$ = density:
  $\rho_f$ = fluid density;
  $\rho_s$ = density of a particle;
$\tau$ = shearing stress;
$\phi$ = a function defined by Eq. 16*a*;
$\psi$ = a function defined by Eq. 16*b*.

# APPENDIX II

## Method of Calculating the Hydraulic Radius in a Cross section with Different Roughnesses

Water flowing through a certain profile continuously transforms its potential energy into heat by means of friction.[9] This friction, along smooth or rough walls, follows certain laws. Formulas expressing these laws contain the velocity of the water $V$, slope of the energy grade line $S$, acceleration of gravity $g$, roughness of the wall, length of wetted perimeter $P$, and the cross-section area $A$. The latter two are commonly combined into the hydraulic radius $R = \frac{A}{P}$. Some of these variables enter the equation explicitly, and others, like $g$ and the roughness, are sometimes hidden in other constants.

In this connection only turbulent flow is of interest. A thin layer of laminar flow exists along the wall, which may or may not be able to hide the irregularities of the wall. In both cases most of the energy of the flow is destroyed by transformation into turbulence—that is, by an irregular deflection of the different flow filaments, caused by instabilities of the laminar layer (smooth wall) or by roughnesses extending out of the layer (rough wall). The filaments (which have been flowing only in the direction of the general flow) are deflected without gaining energy. Therefore, the average velocity in the direction of the flow of the newly established eddy is smaller than the average of the velocities of all the particles composing the eddy prior to its formation. The slow-moving eddy is pulled into the fast flow and accelerated again by other particles by use of part of their flow energy. At the same time, eddy energy is destroyed by laminar friction and the entire process begins over again.

In order to discuss the distribution of friction over a cross section in mathematical terms, two assumptions are necessary. The first assumption is that the entire cross section can be divided into units that will correspond to similar units of the wetted perimeter so that the potential energy of the area unit is transferred into eddy energy along the corresponding surface unit, and these eddies are accelerated and destroyed again in the former area unit. It is evident that the ratio of all area units to the corresponding surface unit will be the same in a cross section with uniform roughness. If the average velocity is assumed to be the same for all units of a simple cross section, every friction formula applicable to the entire section will also be applicable to each unit.

The second assumption is that the same friction formula will be applicable to a certain unit, even though the roughness of other parts of the cross section is changed. This second assumption enables one to make computations on any cross section with composite roughness, as, for instance, a flume with smooth side-walls and granular bed, a river with rough banks or partly sandy and partly rocky bed, etc. The procedure is as follows: For each part ($P_m$) of the wetted perimeter having a certain roughness, one can find the

corresponding area, $A_m$, to be combined into a variable hydraulic radius $R_m = \frac{A_m}{P_m}$. The roughness in this unit may be described by the factor $n_m$. The slope $S$ and the average velocity $V$ are constant for all units. If there are $N$ units, it is possible to establish $N + 2$ equations; that is, $N$ equations of the form—

$$V = f_m(S, g, n_m, \text{and } R_m) \qquad (25)$$

one equation of the form—

$$A = \sum_{m=1}^{N} A_m \qquad (26)$$

which shows that all the area units add up to the total area, and one equation of the form—

$$P = \sum_{m=1}^{N} P_m \qquad (27)$$

which shows that all the wetted-perimeter units add up to the total wetted perimeter.

Consider, as an example, a flume experiment in which the section is divided into two units—bed and side-walls. In this case four equations may be stated. The roughness of the side-walls will be obtained from preliminary experiments. The function $f_w$ and the constant $n_w$ are known. The values of $P_w$, $P_b$, $V$, and $S$ will be measured in each individual experiment. Eq. 27 is meaningless, in this case, because all values of $P_m$ are measured. Eqs. 25 and 26 enable the investigator to compute $A_w$, $A_b$, and $n_b$, if he assumes the function $f_b$ suitable for the character of the bed.

This method can be checked very easily by applying it to a flume with different roughness in the side-wall and a stable bottom. If it is possible to find two roughness factors $n$ that will satisfy measurements having different water depths, the method is satisfactory. The writer has checked the method several times and always has found it to be correct within the accuracy of the measurement. As an example, this procedure is well illustrated when both bed and side-wall friction can be expressed by means of Manning's formula:

$$V = \frac{1.486}{n} S^{0.5} R^{0.67} \qquad (28)$$

As previously stated, $V$, $S$, width $b$, and depth $d$ are measured, and $n_w$ is known; therefore, it is possible to determine $R_w$ directly by using Manning's formula for the wall:

$$R_w = \left( \frac{n_w}{1.486} \frac{V}{S^{0.5}} \right)^{1.5} \qquad (29)$$

The area with reference to the wall is

$$A_w = 2\, d\, R_w \qquad (30a)$$

the area with reference to the bed is

$$A_b = d\,(b - 2\,R_w) \qquad \text{(30b)}$$

and the corresponding hydraulic radius is

$$R_b = \frac{A_b}{b} = d\left(1 - 2\,\frac{R_w}{b}\right) \qquad \text{(31)}$$

By using the radius $R_b$, the roughness factor $n_b$ is then obtained, thus,

$$n_b = \frac{1.486}{V}\,S^{0.5}\,R_b{}^{0.67} \qquad \text{(32)}$$

and the discharge corresponding to the unit width of the bed is

$$q_b = V\,R_b \qquad \text{(33)}$$

The values $R_b$, $q_b$, and $n_b$ have been used in the paper for the evaluation of $\psi$ for all experiments.

To consider a numerical example,[12] let $b = 2.416$ ft; $d = 0.276$ ft; $q = 1.000$ cu ft per sec; and $S = 0.0010$. For turbulent flow[13] the average value of Manning's $n$, applied to the side-walls, is 0.0097. From this information the following computations can be made: $A = b\,d = 0.667$ sq ft; $V = \frac{q}{A} = 1.50$ ft per sec; $R_w = \left(\frac{n_w}{1.486}\,\frac{V}{S^{0.5}}\right)^{1.5} = \left(\frac{0.0097}{1.486} \times \frac{1.50}{0.0316}\right)^{1.5} = 0.172$ ft; $R_b = d\left(1 - 2\,\frac{R_w}{b}\right) = 0.276\left(1 - 2\,\frac{0.172}{2.416}\right) = 0.237$ ft; $n_b = \frac{1.486}{V}\,S^{0.5}\,R_b{}^{0.67} = \frac{1.486}{1.50}\,0.0316 \times 0.384 = 0.012$; and $q_b = V\,R_b = 1.50 \times 0.237 = 0.356$ cu ft per sec per ft of width.

Values $R_b$, $n_b$, and $q_b$ do not contain the influence of the side-wall and, therefore, can be used in any formula for bed-load transportation, the results being expressed for a channel of infinite width.

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[12] Data from "Studies of River Bed Materials and Their Movement, with Special Reference to the Lower Mississippi River," *Paper No. 17*, U. S. Waterways Experiment Station, Vicksburg, Miss., January, 1935, Table 11.

[13] *Loc. cit.*, Tables 8, 9, and 10.

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PAPERS

# DESIGN OF ACCELERATION AND DECELERATION LANES

By Adolphus Mitchell,[1] Assoc. M. Am. Soc. C. E.

Synopsis

Mark Twain once said: "Everybody talks about the weather, but nobody does anything about it." The design of acceleration and deceleration lanes is in a position similar to that of the weather. This paper is an attempt to do something about it. The writer hopes discussers will present data sufficient to preclude argument on some of the controversial factors involved.

## Introduction

The purpose of a deceleration lane is to provide a means for vehicles to leave the highway at cruising speed and come to a stop or a lower speed before entering a street, filling station, or other destination. The purpose of the acceleration lane is to provide a means by which slow-moving traffic can reach the cruising speed of a highway before entering its lanes. The deceleration lane should be long enough to provide for a "weaving distance," motor deceleration to a speed at which braking can be done safely, and a comfortable braking distance from that point on. ("Weaving distance" is used herein to describe the transverse movement from one lane to another whether converging to, or diverging from, the traffic stream.) The acceleration lane should provide for full acceleration to cruising speed, and a weaving distance sufficient for the driver to enter the highway, provided there is a space between cars into which he can turn. In the event no such space is available, or made available by cars on the outside lane moving over, sufficient perception and brake reaction distance should be allowed in addition to a braking distance which would not necessitate pitching the passengers to the floor of the car. A sketch of the elements of the deceleration and acceleration lanes, as proposed, is shown in Fig. 1. The function of the insulation strip is for channelization, to encourage proper use of the speed-change lanes, and to eliminate points of conflict. As will be seen in the illustration, the side entrance to the lanes is tapered so as to

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **July 15, 1941.**

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permit gradual merging with, or diverging from, the traffic stream on the highway.

## Deceleration Lanes

The first element of the deceleration lane as detailed in Fig. 1 is the weaving distance. This distance should be sufficient to allow safe turning at cruising

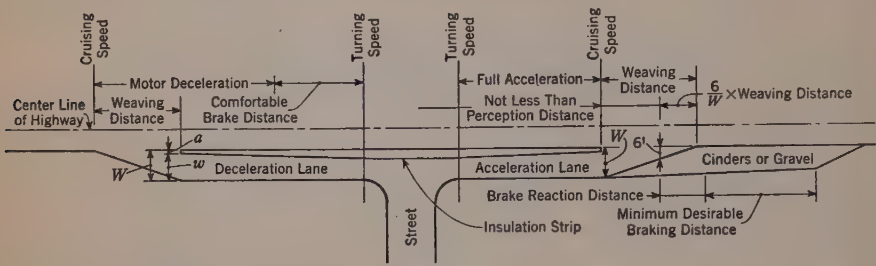


Fig. 1

speed from the highway into the deceleration lane. The curves in Fig. 2 give the weaving distance for any cruising speed with or without an insulation strip. (An insulation strip—see Fig. 1—is any physical barrier separating the deceleration lane from the highway proper. This barrier guarantees correct use of the deceleration lane.) The curves are based upon the formula derived in the Appendix. The width $w$ (see Fig. 1) varies with the speed of the vehicle as follows:

| Speed, in miles per hr | Width, $w$, in ft |
|---|---|
| 60 | 14.6 |
| 50 | 13.8 |
| 40 | 13.0 |
| 30 | 12.2 |
| 20 | 11.4 |
| Minimum | 11.0 |

The assumption of an S-path of travel is a theoretical expedient entirely, and there is no doubt that drivers would find a spiral path more natural than a reverse circular path. The formula derived on the basis of the S-path contains all the variables involved in the natural path. For this reason it is necessary only to determine suitable constants for this formula. Grade is a negligible factor as shown in theoretical studies. Field studies of the weaving distance as measured in terms of time and speed lead to an average coefficient of friction of 0.16. This value is identical with that used everywhere as a comfortable factor for determining the degree of curvature for highways and should not be confused with the maximum obtainable coefficient of friction between tire and pavement. A comfortable coefficient of friction is one that does not make it difficult to keep one's seat while traversing a curve. A comparison of results shown by the curves in Fig. 2 with accepted values of weaving distances is very favorable. Most studies of weaving distances have been made in perfecting the operation of traffic circles where sharp curvatures make the maneuver difficult.

Motorists traveling at high speeds do not like to apply their brakes. This reluctance on the part of the driver must be considered in the determination of the distance required for deceleration, or "slowing" on the highway will result. Values of motor deceleration have been determined by Donald W. Loutzenheiser,[2] Jun. Am. Soc. C. E., for passenger cars on a level grade.

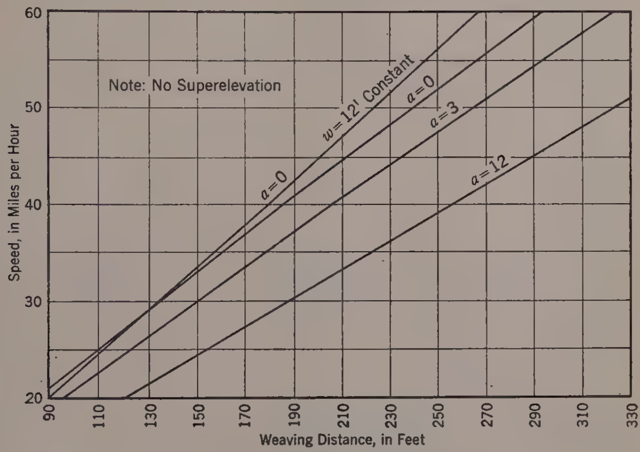


FIG. 2

"Motor deceleration" is the process of slowing the car in gear, with the foot off the accelerator. Fig. 3 gives motor deceleration distances for passenger cars on grades from 3% upgrade to 3% downgrade. These curves are based upon an extension of the results found by Mr. Loutzenheiser. The derivation of the mathematical formulas to include the effect of grade is presented in the Appendix. It was found that motor deceleration from a speed of 40 miles per hr or faster to a speed of 25 miles per hr or less, without resorting to braking, severely taxed the driver's patience. Drivers interviewed by the writer indicated that a preponderance of them think they do not apply brakes until they reach 35 miles per hr or 30 miles per hr, unless it is necessary to do so. Studies, by John Beakey,[3] of normal deceleration from speeds of 50 to 60 miles per hr by the public on approaching a "stop" sign at a highway intersection indicate that there is very little braking at 40 miles per hr, and to all practical purposes substantiate the belief of the persons interviewed. Best results should be obtained by varying the speed to which motor deceleration is assumed with the design speed of the highway (that is, assume 40 miles per hr for a 70-mile-per-hr highway and 30 miles per hr for a 50-mile-per-hr highway). This would be in keeping with the sensation of speed as felt by the driver as well as the time element involved.

The last part of the deceleration lane is that required for a comfortable braking distance from the end of the motor deceleration period to the speed at which it is desired to bring the motorist at the end of the lane. Fig. 4 shows curves for the comfortable braking distance from a given speed to a full stop

[2] "Speed-Change Rates of Passenger Vehicles," by Donald W. Loutzenheiser, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., 1938, p. 90.

[3] "Acceleration and Deceleration Characteristics of Private Passenger Vehicles," by John Beakey *oc. cit.*, p. 81.

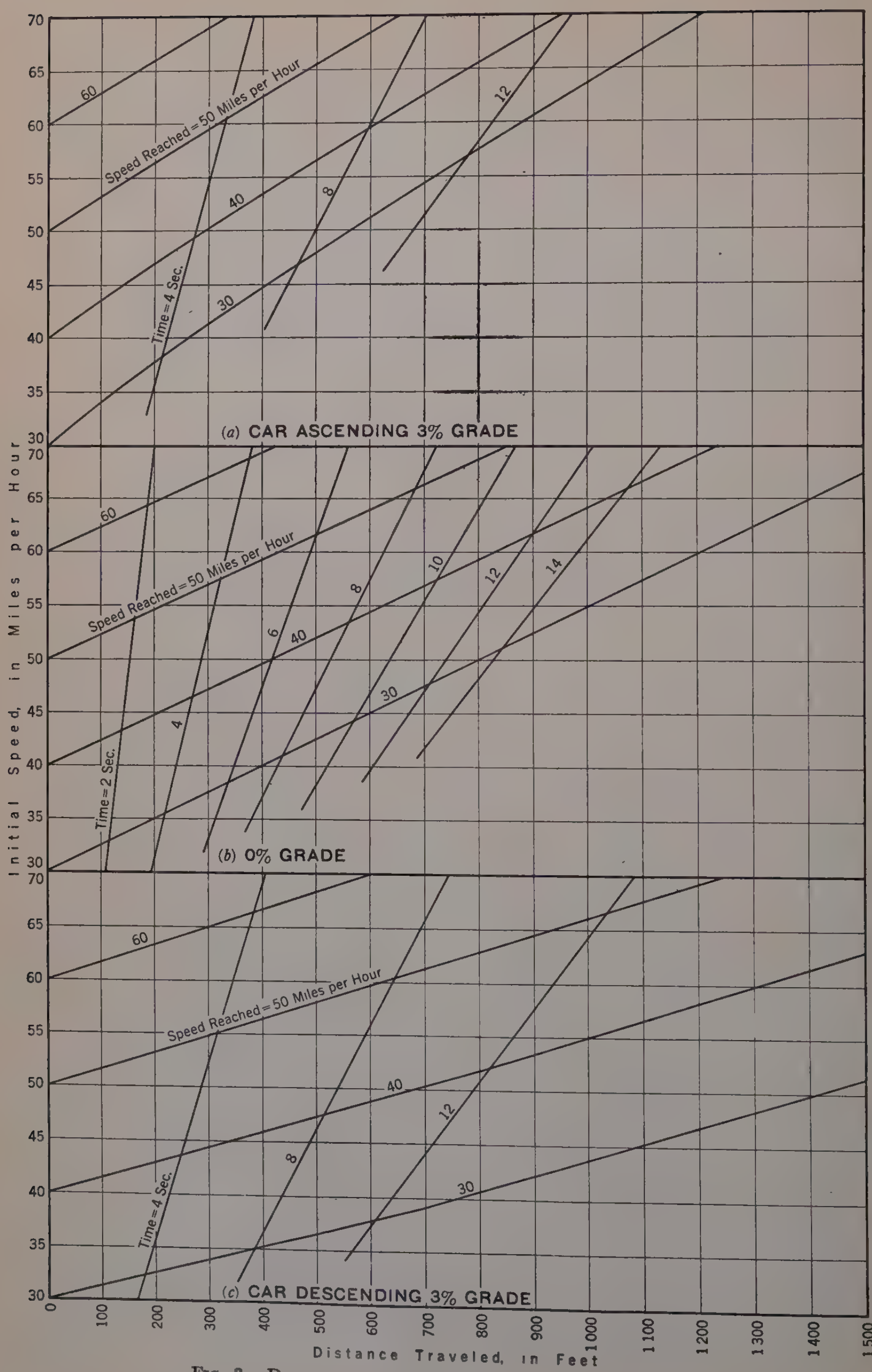


FIG. 3.—DECELERATION DISTANCES FOR PASSENGER CARS

for several grades. These curves are based upon the formulas in the Appendix. The coefficient of friction was assumed to be different for each 10-mile-per-hr variation in speed. It is believed that the motorist senses the stability of his

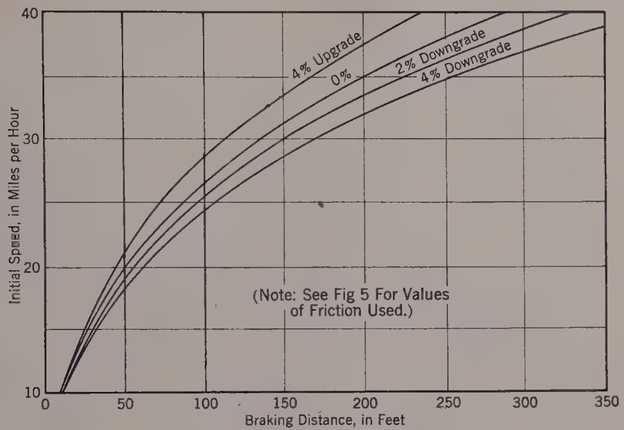


FIG. 4.—COMFORTABLE BRAKING DISTANCE

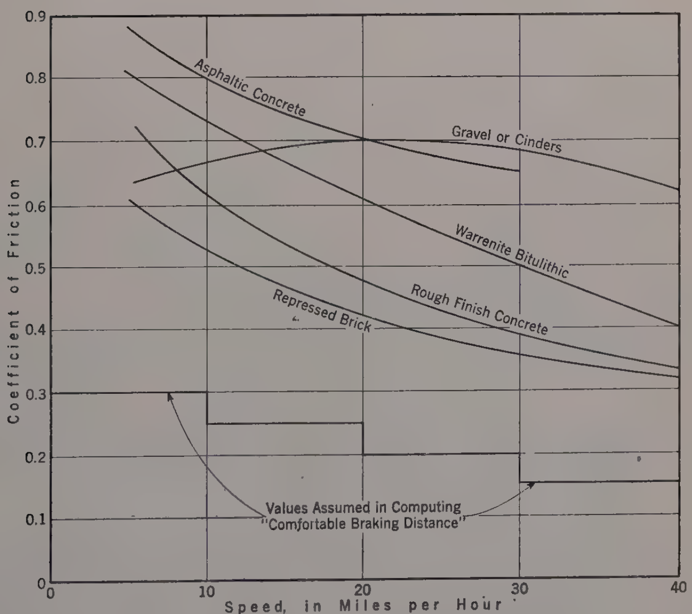


FIG. 5.—COEFFICIENTS OF FRICTION FOR VARIOUS SPEEDS (FULL SKID, STRAIGHT AHEAD, WITH SMOOTH TIRES ON A WET SURFACE)

car and, as the feeling of control increases with a decrease in speed, it is logical to assume that the brake pressure is increased proportionately. In Fig. 5 the coefficients of friction for several surfaces are given, as determined by Ralph

A. Moyer,[4] Assoc. M. Am. Soc. C. E. Mr. Beakey concluded that a rate of deceleration corresponding to a coefficient of friction of about 0.4 is the maximum without discomfort to passengers. As it was desired to use values that would correspond closely to the usual habits of the average driver, it was assumed that a coefficient of friction of 0.15 at 40 miles per hr would increase to 0.3 at 10 miles per hr. The distances indicated in Fig. 4 checked closely with those observed by Mr. Beakey, which substantiate the assumptions made.

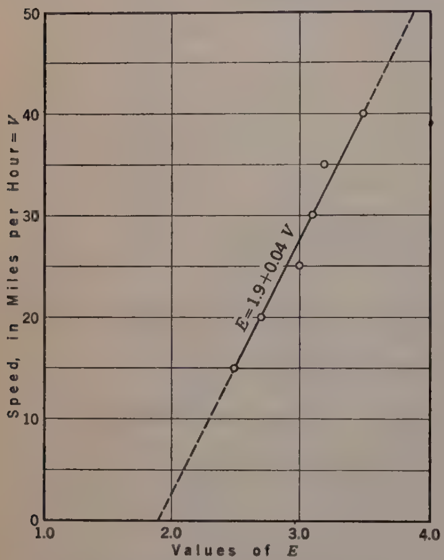


FIG. 6.—AVERAGE DISTANCE, $E$, OF RIGHT FRONT WHEEL FROM EDGE OF 22-FT CONCRETE PAVEMENT, IN FEET

A further refinement of the deceleration lane can be made by funneling. By funneling is meant to provide a decreasing width of lane as the speed decreases. Operators will not drive as fast on narrow as on broad pavement. J. T. Thompson and Norman Hebden[5] determined the average distance of the right front wheel from the edge of the pavement for speeds from 40 miles per hr to 15 miles per hr. Fig. 6 shows results of their observations with an extrapolation sufficient to cover most requirements. In this curve,

$$E = 1.9 + 0.04\,V \ldots\ldots (1)$$

is the average distance, in feet, of the right front wheel from the edge of a 22-ft concrete pavement. Until better information is available, it is suggested that these results be used as a basis for the variation in width of deceleration lanes, provided the minimum is kept at 11 ft.

To illustrate the application of the foregoing fundamentals to the approach of a filling station, attention is directed to Fig. 7. The cruising speed of the highway is assumed to be 50 miles per hr. Entering Fig. 2, it is found that a weaving distance of 265 ft will be required if a 3-ft insulation strip is used. The car is going up a 3% grade as it approaches the filling station, and (see Fig. 3(*a*)) a distance of 550 ft will be required for motor deceleration to a speed of 30 miles per hr, at which speed brakes are assumed to be applied. From Fig. 4 it is found that 120 ft would be required for a comfortable braking distance to a full stop. The 50-ft radius of the turn-in will permit a 15-mile-per-hr speed very comfortably. Therefore, it is not necessary to bring the motorist to a complete stop. From Fig. 4 it is found that the comfortable braking distance from a speed of 15 miles per hr to a complete stop is approxi-

[4] "Skidding Characteristics of Automobile Tires on Roadway Surfaces, and Their Relation to Highway Safety," by Ralph A. Moyer, *Bulletin No. 120*, Iowa State College Eng. Experiment Station, Ames, Iowa, 1934.

[5] "A Study of the Passing of Vehicles on Highways," by J. T. Thompson and Norman Hebden, *Public Roads*, U. S. Dept. of Agriculture, Bureau of Public Roads, 1937, pp. 121–137.

mately 20 ft. The distance required for comfortable braking from a speed of 30 miles per hr to one of 15 miles per hr, therefore, is 120 − 20 = 100 ft. In this manner, and with these assumptions, it is found that a total distance of 650 ft is required to bring a car from a cruising speed of 50 miles per hr to a turning speed of 15 miles per hr.

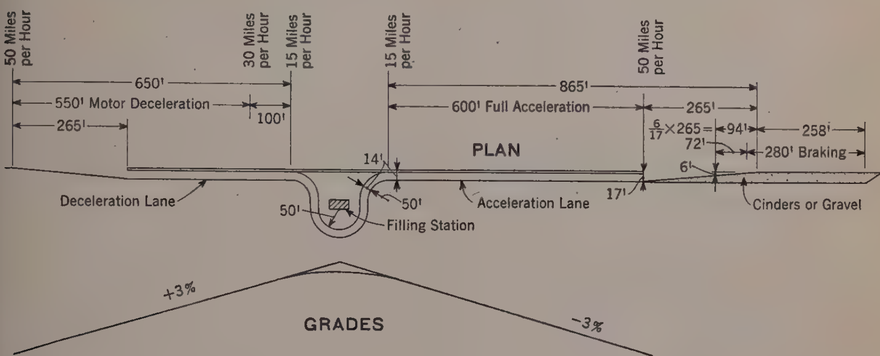


Fig. 7

## Acceleration Lanes

Acceleration lanes may be divided into a distance for full acceleration to cruising speed, plus a weaving distance to permit entering the traffic lane. In case it is impossible to find an opening in the highway traffic, it will be necessary to consider the provision of perception distance, brake reaction distance, and a braking distance. In order to encourage acceleration, it is desirable to increase the width of the lane as the speed increases. The use of an insulation strip is advisable to make certain that the drivers cannot enter traffic in less than a distance sufficient to eliminate dangerous speed differentials.

Values of full acceleration distance were determined by Mr. Loutzenheiser.[6] He defined full acceleration as a maximum pickup possible under the control of typical drivers. The model cars tested varied from 1935 to 1937, and tests were made on flat grades. An extension of these results to include grades from 3% upgrade to 3% downgrade is given in Fig. 8. These curves are based upon the mathematical formula derived in the Appendix.

The weaving distance is the same as that for the deceleration lane. (In the treatment of acceleration and deceleration lanes, the weaving distance has been assumed the same whether for cars diverging from, or converging with, the traffic stream. Mathematically, the diverging weaving distance could be slightly smaller, provided that the accelerator was released on leaving the traffic lane, but, since the decrease in speed would be so small, no difference of practical importance would result.) It is further stipulated, however, that the weaving distance should not be less than perception distance plus a distance depending upon the ratio of the width of a car to that of the acceleration lane at the beginning of the weaving distance, as shown in Fig. 1.

Perception distances can be read directly for any speed from Fig. 9(*a*).

[6] "Speed-Change Rates of Passenger Vehicles," by Donald W. Loutzenheiser, *Proceedings*, Highway Research Board, National Research Council, Washington, D. C., 1938, p. 92.

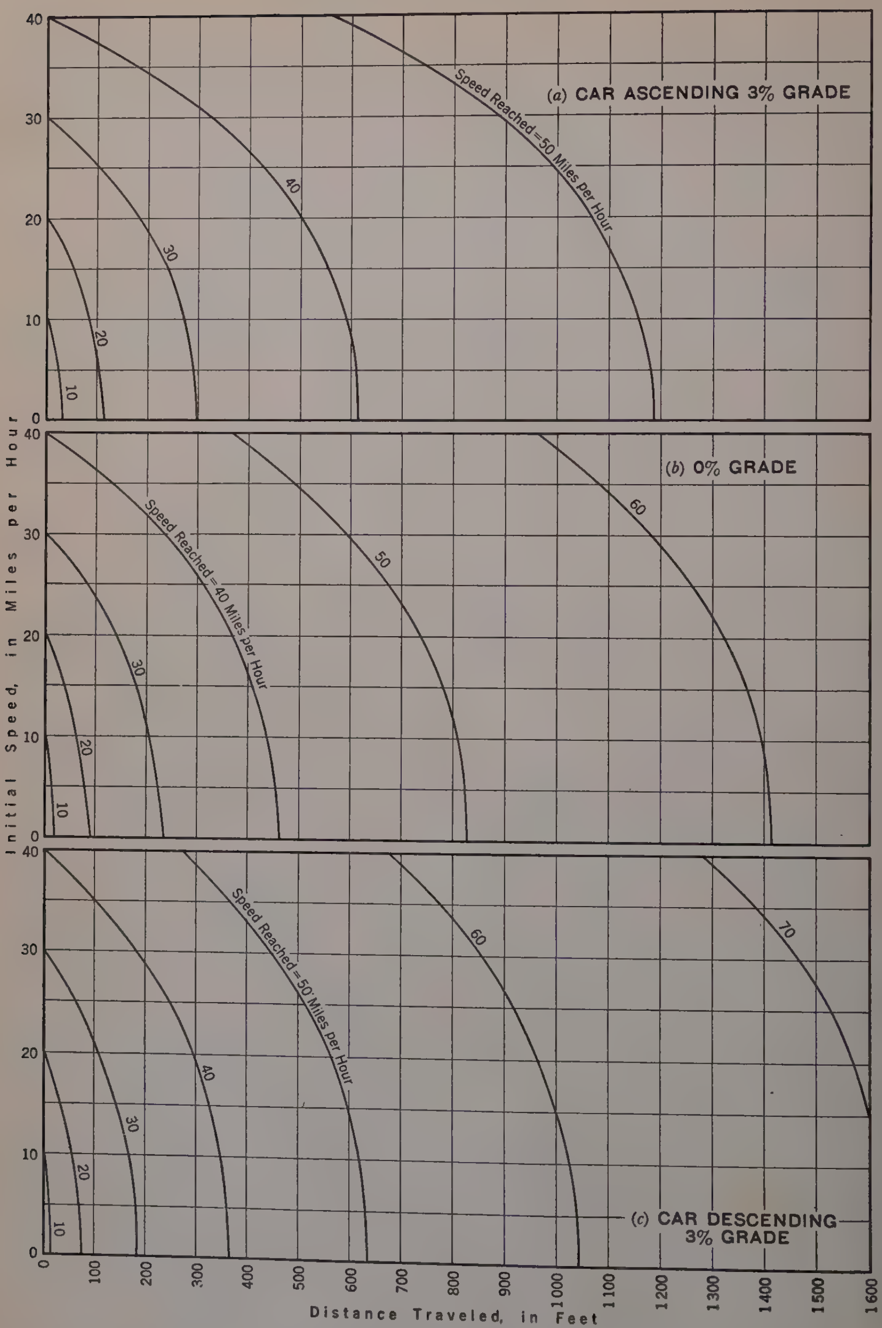


FIG. 8.—ACCELERATION DISTANCES FOR PASSENGER CARS

Perception distance is the product of the time in seconds it takes a driver to determine whether or not he can merge with the cruising lane, multiplied by the speed of his car, in feet per second. The average driver will probably complete this perception in less than one second, provided he has previously appraised the situation at the point of entry. Therefore, the length of the taper at the end of the acceleration lane should not be less than perception distance plus the ratio of width of car to width at the throat of the acceleration lane, multiplied by the weaving distance as given in Fig. 2. The values taken from Fig. 2 will always be greater than the minimum requirements previously outlined.

The brake-reaction distance is based on a time of one second. The distance for any speed may be read directly from Fig. 9(*a*). This is the distance required for the driver to move his foot from the accelerator and depress the brake.

The braking distance may be read directly from Fig. 9(*b*) for any speed. This is the minimum desirable braking distance required to bring a car to a full stop. As previously stated, 0.4 is the maximum value of the coefficient of

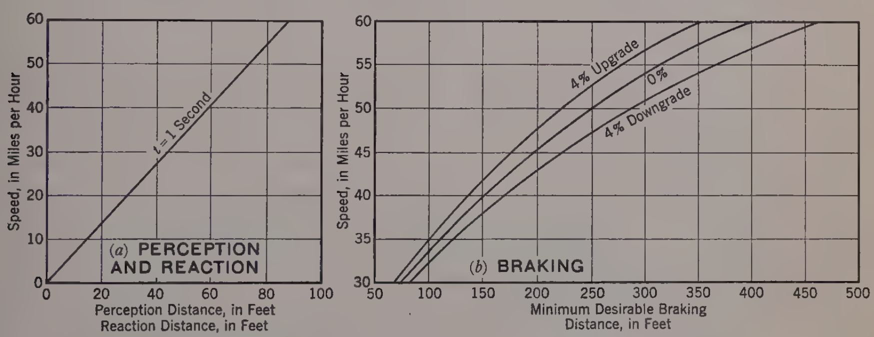


FIG. 9.—PERCEPTION, REACTION, AND BRAKING DISTANCES

friction that can be used without discomfort to passengers. A variable value for the coefficient of friction was adopted beginning with the value of 0.25 at 60 miles per hr and increasing to a value of 0.4 at 30 miles per hr.

To illustrate the application of the foregoing fundamentals to the exit of a highway filling station, attention is again directed to Fig. 7. It is assumed that the car enters the acceleration lane at a speed of 15 miles per hr, and is heading down a 3% grade. Referring to Fig. 8(*c*), it is found that 600 ft will be required to attain a cruising speed of 50 miles per hr. In Fig. 2 it is found that a weaving distance of 265 ft will be required. If it is impossible for the accelerating car to enter the traffic lane, the driver will realize this by the time his car reaches that part of the tapered end which is 6 ft in width. The average car is about 6 ft wide, and to travel on a part of the taper less than this width would mean entry on to the traffic lane. This point is found, by similar triangles, to be 94 ft from the end of the weaving distance. From this point, a brake-reaction distance of 72 ft is provided as called for in Fig. 9(*a*). The minimum desirable braking distance as given in Fig. 9(*b*) is 280 ft. Thus it is determined that an 865-ft concrete apron is required for the acceleration lane. It is found further that an additional 258 ft of the improved shoulder should be provided for emergency stops.

### Justification

The provision of adequate acceleration and deceleration facilities at the intersection of a high-speed with a low-speed roadway or turn may be justified by accident experience. Of the accidents on the Schuyler Merritt Parkway, in Connecticut, 71% have been due to internal stream friction. Most of these were further classified as rear-end collisions. Since actual trials indicate that many of the exits and entrances to this highway permit speeds no higher than 20 miles per hr, there remains little doubt as to the location of a sizable percentage of the rear-end collisions, although locations given on accident reports have been too indefinite to be positive in this matter. In cases of this kind, which involve state highway intersections and state-owned roadside parks, there is manifestly a responsibility falling on the state to provide adequate facilities.

The entrances and exits of private property and small business establishments present a problem. It has been estimated that there is a filling station for each $1\frac{1}{2}$ miles of highway. The cost of providing adequate acceleration and deceleration lanes would be several times the value of most of these stations. The problem is even greater when all the eating houses, open-air theaters, etc., are added to the filling stations. Regulation by the state department of traffic control can do much to improve existing conditions on the basis of cooperation between the state and the individual, but the complete solution advocated in this paper appears to be economically justifiable only where the right of access of the abutters is purchased along with the right of way, as has been done in the building of parkways and freeways. It has been predicted that the next decade will bring 25,000 miles of new parkways and freeways, which in itself promises a sizable interest in the problem of speed-change lanes. Once the abutter has no right of access, it is possible to space filling stations so as to leave no doubt of the warrants for the provision of correctly designed acceleration and deceleration lanes.

### Acknowledgments

The writer wishes to express his appreciation to all members of the staff of the Yale Bureau for Street Traffic Research for their encouragement and sound suggestions. Special credit is due Leslie Williams for his painstaking assistance in improving the manuscript.

### Conclusion

It is concluded that acceleration and deceleration lanes can be reduced to component parts that can be individually analyzed and determined with sufficient accuracy for design purposes. Existing highways have not been provided with adequate acceleration and deceleration lanes. Failure to make such provision has necessitated deceleration to a low speed in the main traffic lanes. As speeds have increased, accident records have disclosed the danger in creating a large differential in speed between traffic on the highway and traffic in the process of leaving or entering it. In designing entrances and exits for an express highway, the principal pitfall to be avoided is insufficient lengths. The motorist will not leave the highway at cruising speed if he knows

it will be necessary to bring himself to a violent stop on a deceleration lane that is much too short. He cannot avoid dangerous differentials in speed if the acceleration lane is too short.

## APPENDIX

### Derivation of Formulas

*Weaving Distance.*—From geometry, "the sum of the squares of the legs is equal to the square of the hypotenuse"; hence (see Fig. 10),

$$\left(\frac{L}{2}\right)^2 + \left(r - \frac{W}{2}\right)^2 = r^2 \quad \text{..(2a)}$$

or

$$L = \sqrt{W\,(4\,r - W)} \quad \text{..(2b)}$$

Standard texts develop the formula for superelevation:

$$r = \frac{V^2}{15\,(f + e)} \quad \text{....(3)}$$

in which: $V$ = speed in miles per hour; $f$ = coefficient of friction; $e$ = superelevation in feet per foot; and $r$ = radius in feet.

The edge clearance,[5] in feet, should be equal to $1.9 + 0.04\,V$ (see Fig. 6); hence

$$W = a + b + 3.8 + 0.08\,V \quad \text{.(4)}$$

in which: $a$ = width of insulation strip in feet; and $b$ = width of car in feet.

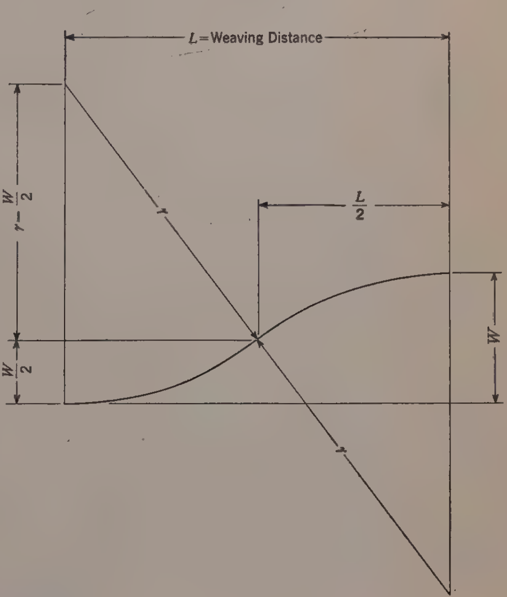


Fig. 10

*Motor Deceleration Formulas.*—From Newton's Second Law of Motion and Fig. 11, the effect of grade on motor deceleration is:

$$\text{Deceleration} = -\,g\,\frac{G}{100} \quad \text{......(5)}$$

in which $g$ = acceleration of gravity in feet per second² and $G$ = per cent grade. The motor deceleration[2] is as shown in Fig. 12 for flat grades; hence

$$\text{Motor deceleration} = -\,a = \frac{3}{100}\,(v + 15) - g\,\frac{G}{100} \quad \text{......(6)}$$

Integrating and converting $v$ to miles per hour:

$$\left(V + 10 - g\,\frac{G}{4.5}\right) e^{(3/100)t} = \bar{V} + 10 - g\,\frac{G}{4.5} \quad \text{......(7)}$$

in which $t$ = time in seconds; $\bar{V}$ = initial speed; and $V$ = final speed. Integrating again, the deceleration distance is

$$s = 50\left[-\left(10 - g\frac{G}{4.5}\right)\log_e\left(\frac{\bar{V} + 10 - g\dfrac{G}{4.5}}{V + 10 - g\dfrac{G}{4.5}}\right) + (\bar{V} - V)\right] \quad \ldots\ldots(8)$$

in which $G$ changes sign to minus for uphill travel.

*Braking Distance.*—Let $G$ = per cent grade; $F$ = force in pounds; $\bar{V}$ = initial speed in miles per hour; $V$ = final speed in miles per hour; $s$ = braking

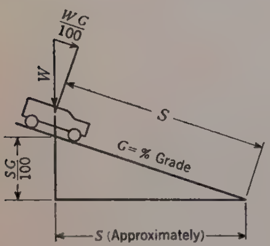


FIG. 11.—EFFECT OF GRADE

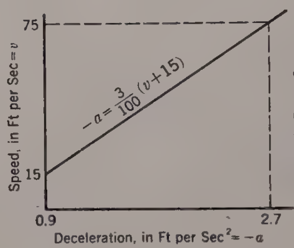


FIG. 12

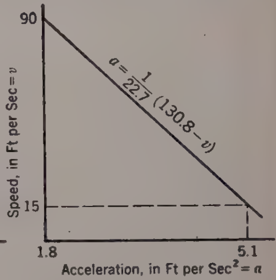


FIG. 13

distance in feet; $W$ = weight of vehicle in pounds; $g$ = 322 ft per sec²; and $f$ = coefficient of friction. From energy conservation:

$$F \times s = \frac{1}{2}\frac{W}{g}(1.47)^2(\bar{V}^2 - X^2) + \frac{W\,G}{100}s = \frac{W}{g}(f\,g)\,s \quad \ldots\ldots(9)$$

For downhill travel (solving Eq. 9 for $s$):

$$s = \frac{\bar{V}^2 - V^2}{30f - 0.3\,G} \quad \ldots\ldots(10a)$$

and, for uphill travel:

$$s = \frac{\bar{V}^2 - V^2}{30f + 0.3\,G} \quad \ldots\ldots(10b)$$

*Full Acceleration.*—From Newton's Second Law of Motion and Fig. 11, the effect of grade on full acceleration is:

$$\text{Acceleration} = g\frac{G}{100} \quad \ldots\ldots(11)$$

Full acceleration[2] is as shown in Fig. 13 for flat grades; hence,

$$\text{Full acceleration} = a = \frac{1}{22.7}(130.8 - v) + g\,G \quad \ldots\ldots(12)$$

Integrating twice, the acceleration distance is:

$$s = 34.05\left[(87.2 + 0.1513\,g\,G)\log_e\frac{87.2 - \bar{V} + 0.1513\,g\,G}{87.2 - V + 0.1513\,g\,G} - (V - \bar{V})\right] \ldots(13)$$

In Eq. 13, $G$ is positive for downhill travel and negative for uphill travel.

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PAPERS

# MISSOURI RIVER SLOPE AND SEDIMENT

By William Whipple, Jr.,[1] Assoc. M. Am. Soc. C. E.


## Synopsis

The project for the improvement of the Missouri River consists primarily of open-channel regulation, which contracts the natural channel in addition to materially changing its shape. A general description of the methods adopted is given, together with a quantitative summary of the effects of the improvement upon the length, slope, width, shape, discharge, velocity, and roughness coefficient of the natural stream between Rulo, Nebr., and Sioux City, Iowa. Data are supplied as to the bed and suspended sediment characteristics of the river, in both improved and unimproved sections. An analysis is presented of the applicability of various bed-load formulas, involving both competence and capacity, to the prediction of the future slope of the river; and results are compared with observations to date (1940) on completed sections of the river. It is generally concluded that: (1) Formulas involving competence will not give the answer to this particular problem; (2) the mean slope of the Missouri River eventually will decrease through the operation of the contraction works; and (3) the bed of the river will scour out progressively for some time to come.


## Introduction

The improvement of the Missouri River consists of open-channel regulation of approximately the lower 760 miles of the river, with regulation of flows through the Fort Peck Reservoir (see Fig. 1). It has been prosecuted by the Corps of Engineers primarily in the interest of navigation since authorization by Congress in 1927 and 1935, but some work was done under earlier authorities dating back to 1876. Despite the magnitude of this project and the length of time since it was begun, comparatively little has been published regarding the hydraulic characteristics of the stream. As far as the writer is aware, this paper represents the first published compilation of the basic data from which a prediction can be made of the ultimate effect of this improvement upon the

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by July 15, 1941.

[1] Captain, Corps of Engineers, U. S. Army, Fort Belvoir, Va.

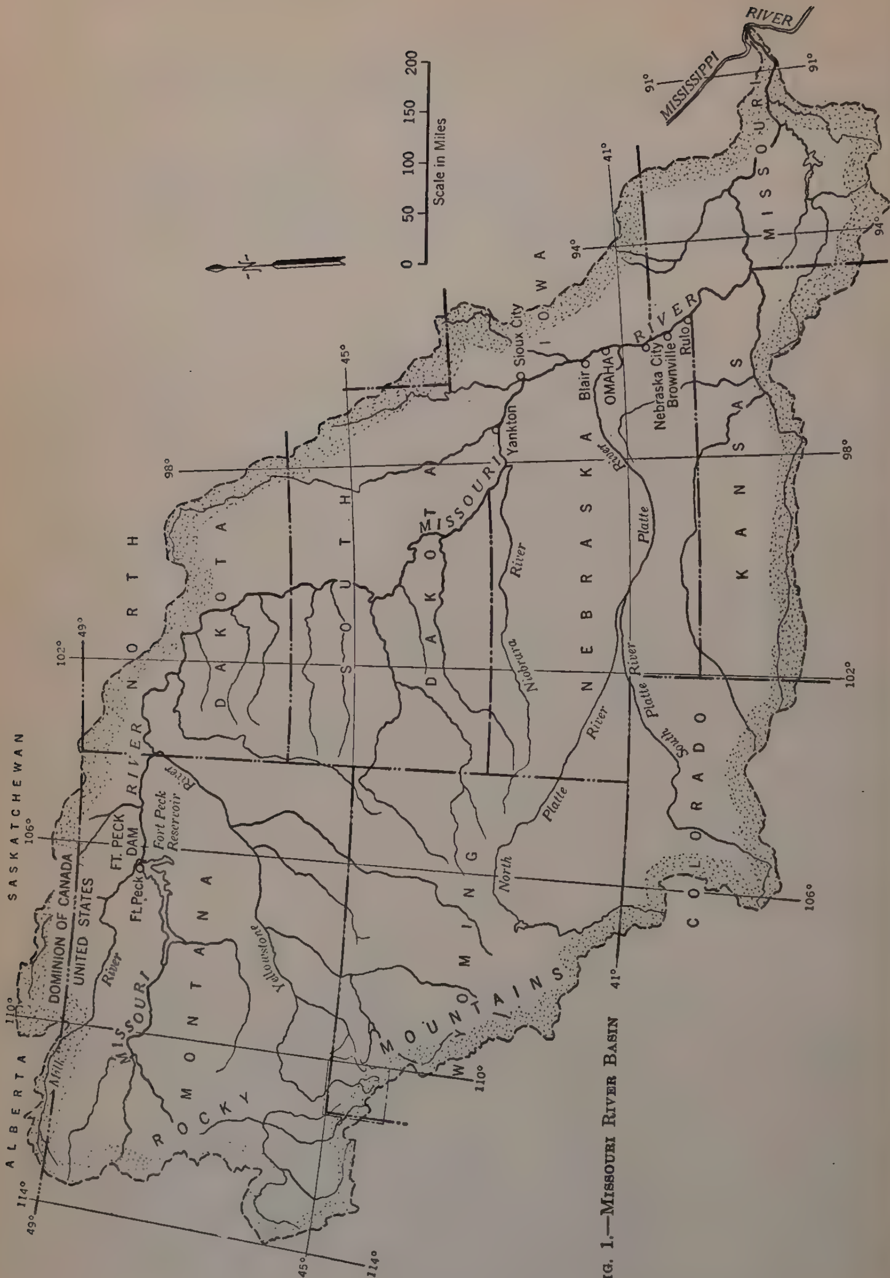


Fig. 1.—Missouri River Basin

hydraulic characteristics of the river. The principal phenomena involved are governed by channel erosion and deposition, and the present study is concerned mainly with the effect of the improvement works upon the competence and capacity of the stream and the probable effect of these changes upon its eventual slope. Data presented deal principally with the part of the river between Sioux City and Rulo, which is approximately the upper third of the improvement, but the results obtained should have some application to the lower sections of the river, as methods used have been generally similar.

## General Description

A complete description of the Missouri River and its tributaries has been published in the "308 report" on the Missouri River.[2] Certain of the applicable basic data are summarized herein. The Missouri River (see Fig. 1) is about 2,470 miles long and drains an area of 529,000 sq miles. The headwaters of the Missouri River and some of its tributaries lie in the Rocky Mountain province, but the greater part of the drainage basin lies in the Great Plains province and the central lowlands. It is noteworthy that, within the area improved, the Missouri River runs through and over deposits of its own sediment, which fill an ancient valley much deeper than the present stream bed at all points. The river never strikes rock or other unerodible material except at occasional points where it approaches the adjoining bluff. The Missouri River experi-

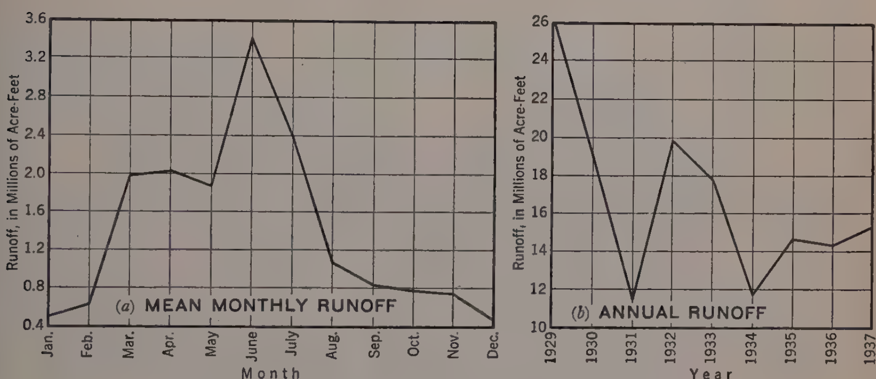


Fig. 2.—Runoff Data, Missouri River at Omaha, Nebr.

ences wide fluctuations in discharge. Fig. 2 shows the mean monthly runoff and the total annual runoff for the years 1929 to 1937, inclusive, at Omaha, Nebr. The year 1938 was excluded because in that year partial operation of Fort Peck Reservoir changed conditions materially from what they would naturally have been.

## Nature of Improvement

The improvement of the Missouri River was based upon the assumption that its average slope represented approximately a state of equilibrium under

[2] *H. R. Doc. No. 238*, 73d Cong., 2d Session, February 5, 1934.

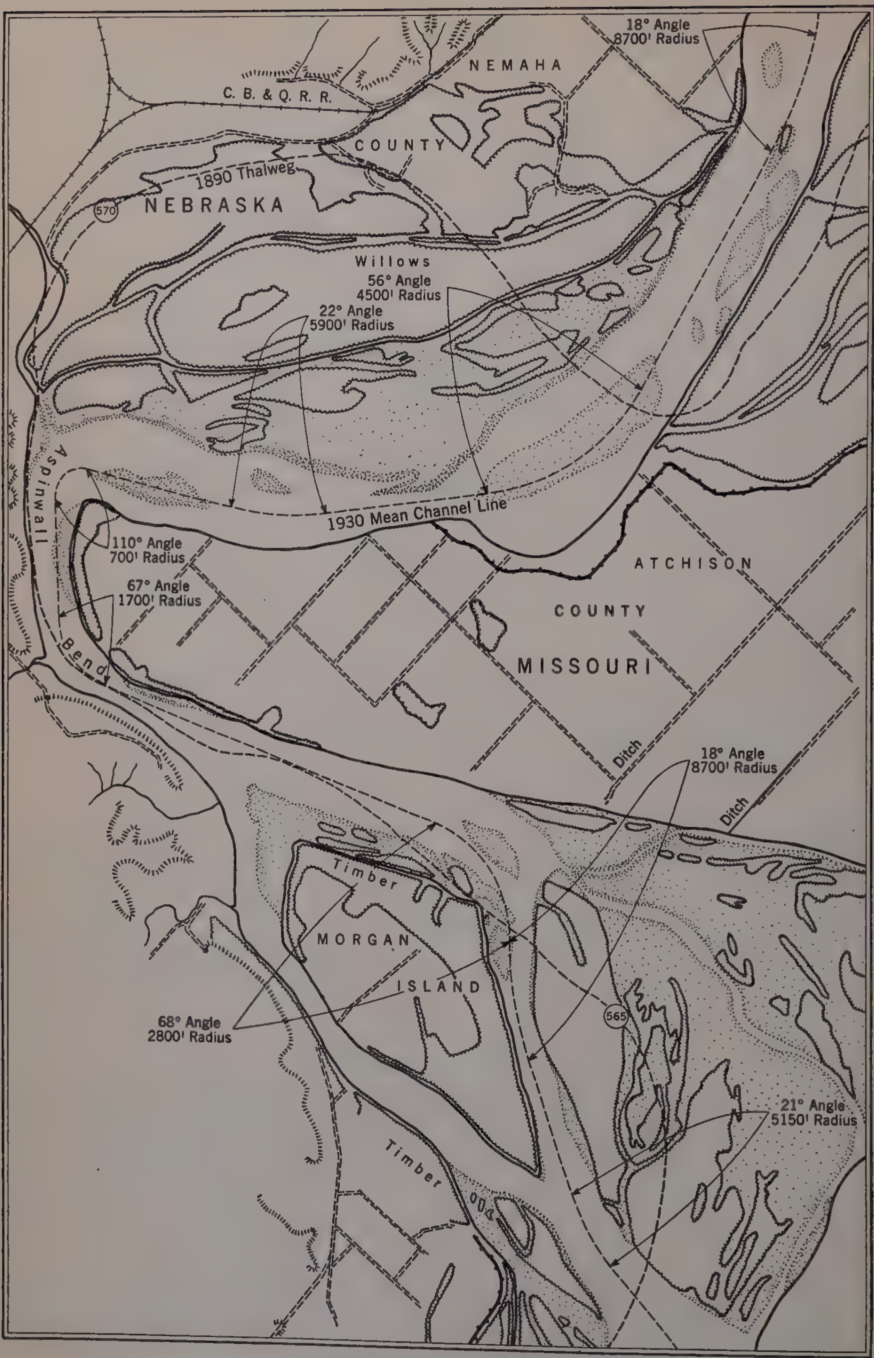


FIG. 3.—ASPINWALL BEND, 1930

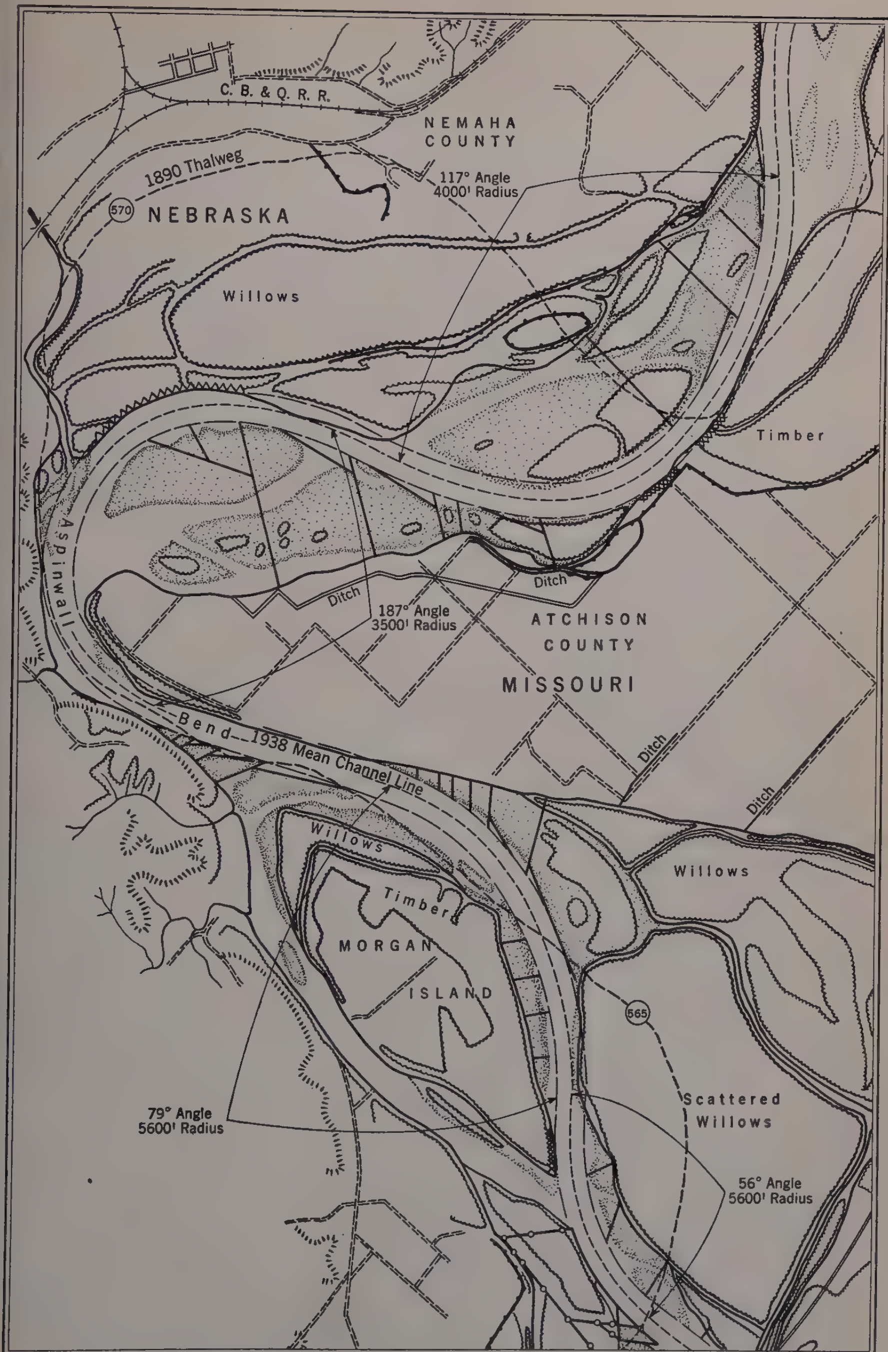


Fig. 4.—Aspinwall Bend, 1938

conditions existing at that time. The sediment content of the Missouri River is enormous, and if the existing slope had been insufficient to carry the average load for a long period of time the valley would have aggraded until an equilibrium slope was reached. On the other hand, it was obvious, from the many meandering oxbow lakes and the low banks, that below Sioux City the Missouri River was not in process of eroding a deeper channel. Although the river in its natural state is constantly varying its length and slope locally, these changes tend to counterbalance one another. It was known that during periods of prolonged drought the river generally silted up its bottom at all points, but these effects were not permanent. The net result of changes for several decades prior to the improvement program was an increase in slope; but insufficient information is available to determine whether, under conditions of nature, this increase would have continued, remained static, or reverted to its former level.

*Method of Improvement.*—The plan for improving the river includes a generally contracted channel in a series of easy curves rather than the succession of straight, wide reaches and involved meanders or abrupt changes in direction that had existed previously. The channel decided upon lay as far as possible within the existing channels of the river. Figs. 3 and 4 illustrate the improvement of one of the worst bends in the river.

When it is determined that the river is to be driven away from a given bank a system of permeable pile dikes is constructed, leading away from this bank. Such a system reduces the velocities in the areas selected for deposit and increases the velocities in the areas selected for the regulated channel. Consequently the water passing through the dikes deposits its heaviest sediment, while the water passing around the dikes, with the increased velocity, scours the bed and bank and erodes that part of the channel deeper and wider. As the erosion and deposition process continues, the dikes and areas between the dikes are silted in and the channel is secured.

As the channel, deflected by the completed dike system, erodes back to its designed alinement on the concave side, this bank is revetted and permanently stabilized, progressively as it reaches the designed alinement. The new bars formed on and over the dike system quickly grow up in willows, which further aid the silting process, so that, a few years after construction is initiated, the new channel is entirely within the designed alinement, with no obstruction to flow except the ends of a few dikes that have not yet completely silted in. Regular snagging operations keep the channels free from obstruction. In addition to the dikes and revetment, dredging is used to cut away deposits of clay or gumbo, to cut pilot channels through bars and islands, and to make cutoffs. The first such cutoff on the Missouri River was described in 1938 by Lieut.-Col. W. M. Hoge.[3]

This river improvement, although constructed primarily in the interests of navigation, will operate to stabilize the banks of the Missouri River, permanently, thus protecting adjacent farm lands and other improvements that were formerly in continuous danger from the ever shifting channel. Since the Missouri River is a navigable waterway, land ownership extends only to the low-water line; and when all of an original holding has once been taken by

[3] *Engineering News-Record,* December 15, 1938, p. 755.

erosion, the owner no longer holds a valid title, even though land might be reformed by accretion in the same place the following year. However, when the channel shifts from one side of the holding to the other through the process of avulsion, the title remains in the original owner and the jurisdiction in the original state. The shifts of the river channel have been so numerous and intricate that at many points land known originally to have been in Iowa now lies on the Nebraska bank, and vice versa; and for practically all land adjacent to the river no conclusive determination of either state or private boundaries has been possible.

*Length.*—The Missouri River originally flowed in a changing course involving successions of straight reaches, followed by abrupt and sometimes meandering bends. The improved river will have sinuous bends in place of all straight reaches but several of the worst bends will have been removed by cutoffs. There is considerable difficulty in stating the exact length and slope of a given part of the Missouri River, as the length of channel and the fall in the water surface between any two points varies not only from year to year but with the stage of the river. The length of the channel of the Missouri River between Rulo and Sioux City was measured (1890) as 270 miles. The length of the center line of the stabilized project channel between the same points will be 245.6 miles. In 1930, shortly before the improvement works began, the length of the thalweg at low stages was 251.8 miles, but at medium stages the length of the mean channel was almost the same as that of the present project channel.

*Mean Slope.*—With regard to the difference in water-surface elevations between two points on the river, the same difficulty exists, although the differences are proportionately smaller. The maximum water-surface elevation reached during the 1927 flood was about 240 ft higher at Sioux City than at Rulo, and for the 1938 flood, about 234 ft. However, a uniform low-water plane, established in 1938, shows a difference of 236 ft, which will be used as the average head available to the river between these two points. This corresponds to a mean slope of 0.96 ft per mile, at medium stages, as long as the head available remains unchanged.

*Width.*—The mean width of the project channel above Rulo will be 725 ft, with a maximum of 1,000 ft and a minimum of 700 ft. The latter will prevail throughout the greater part of this section of the river. The 1930 channel had minimums considerably less than 1,000 ft, a mean usable width of 3,650 ft, and a maximum width not easily measured because of the many islands, back-chutes, and low bars that are difficult to classify.

*Shape.*—It is obvious that major changes in shape and curvature of a river may greatly influence its regimen. The new project channel will be fairly uniform and relatively easy to classify and describe, but the old channel was so irregular in shape that it is difficult to present any idea of its hydraulic characteristics other than by a complete series of large-scale maps much too bulky for presentation in this paper. Back-chutes, split channels around islands, and small bars, with or without vegetation, were very numerous in the uncontrolled river, but it is difficult to describe this condition other than qualitatively and by the example afforded by Fig. 3, which covers only a very small part of the river.

Although the eventual slope of the Missouri River depends primarily upon its sediment-carrying characteristics, certain hydraulic conditions must be evaluated as a basis for the direct approach to the problem. When the improvement of this section of the Missouri River is completed, it will have contracted the channel to a uniform width, left the length unchanged, removed snags, eliminated split channels, generally changed the shape of the channel, substituted dikes and revetment for natural banks, and regulated natural flows through operation of Fort Peck Reservoir. These factors, and other factors, not part of the improvement, all have their effect upon the general hydraulic characteristics of the river, which in turn determine its sediment-carrying capacity; and these relationships are evaluated in this paper to the extent that the available data will permit.

## General Hydraulics

*Local Slope Variations.*—The existence of a substantial uniformity in mean slope taken at various times must not be understood as implying a uniformity of slope over different parts of the river at any one time. Gages set at 5-mile intervals show very considerable variation. Studies of water-surface elevations at gages established at half-mile intervals or less, with precautions to avoid discrepancies due to superelevation, show very great local variation in slope. Even for fairly well stabilized parts of the river, local slopes still vary from less than 0.5 to more than 2.5 ft per mile. Moreover, a new profile taken at the same point a few weeks later may show considerable change.

A study of all available data sheds light on certain slope irregularities and leaves others not yet explainable. For example, two persistent steep slopes were identified as due to the presence of protruding ledge rock into the channel from the adjacent bluff. Some longer reaches, the slopes of which are greater than average at high water only, unquestionably demonstrate disturbing effects of excessively severe bends. In incompleted parts of the river certain slope deviations in both directions from the mean are known to be incidental to the improvement works, and therefore of temporary nature. Sufficient data have not been developed to show the extent of local steepening to be expected at crossings, partly because studies of this nature have not yet been made at extremely low stages and partly because of the presence of so many other disturbing influences of greater magnitude. The effects upon the slope of the Missouri River of the heavy bed load carried by the Platte River have previously been described in detail elsewhere.[4] These effects are of major importance and extend many miles down the river. As measured during the low water of 1930, the mean slope for 31 miles above the mouth of the Platte River was 0.74 ft per mile, and for 44 miles below that point it was 1.24 ft per mile. Locally, steep slopes are found at many points where back-chutes reenter the main stream at an angle, whether or not this occurs at a crossing. This is probably due to bar deposition in the main stream by the chute during times of high water. These half-mile gages have been set only on improved and partly improved reaches of the stream, but similar or greater variations are believed to exist in unimproved sections of the river.

[4] *H. R. Doc. No. 238*, 73d Cong., 2d Session, February 5, 1934, Appendix XV.

It is probable that the completed channel will always have certain slope irregularities, due to tributary influences, variations in shape, crossings, and irregularities in the transportation of bed load. However, disturbances at reentrant back-chutes will be eliminated. The extent of excessive slopes at crossings and bends will vary according to the number and severity of bends.

*Comparison of Shape.*—This study was based upon complete large-scale maps of 1930 and 1939, which show, respectively, conditions prior to any improvement in the section between Sioux City and Rulo and conditions of the present, including the adopted channel line. As a first step in classifying bends, lines were drawn on each map to show, as accurately as possible, the mean channel line of the river at medium stages throughout its length. For the improved channel this was taken to be the center of the project channel. For the maps of the unimproved river it was necessary to disregard small irregularities, and in the case of a split channel either to adopt a line intermediate between the two parts or to follow either one of the channels that was considered to be representative, in both length and curvature, of the river as a whole. Figs. 2 to 4 illustrate these lines for the reaches of the river concerned.

Although excess loss of energy in a bend is occasioned primarily by sharpness of curvature (measured by length of radius in the bend) it is also affected (particularly during overbank flow) by the total angle through which the river is turned. In the many bends of varying radii only the minimum radius in each bend was considered. The total angle through which the river is turned in a bend will be known herein (for lack of better nomenclature) as a "central

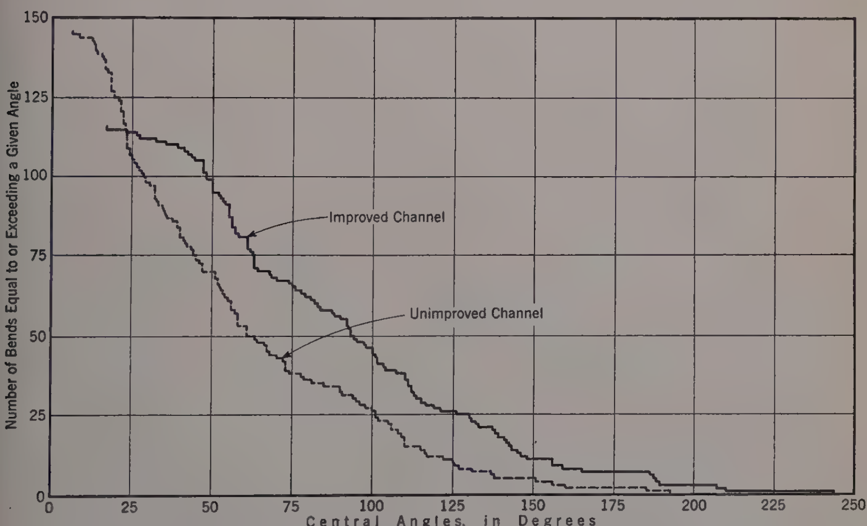


FIG. 5.—NUMBER OF RIVER BENDS CLASSIFIED BY THEIR CENTRAL ANGLES

angle." For convenience, all bends of less than 45° central angle were considered separately, as experience shows that, even during overbank flow, relatively little disturbance is caused by flow through bends of small central angle, despite radii less than those ordinarily desirable.

Fig. 5 shows, for both the improved and the unimproved channel, the number of bends with central angles equal to or greater than any given curvature. These curves indicate that, apparently, the channel stabilization program has increased the number of bends of a central angle of 100° or more, but has made little change in the number of those between 45° and 100°. Actually the number of the longest meanders was reduced by cutoffs, but this fact is not apparent from Fig. 5 because many meanders in the unimproved river had to be classed as two or more successive bends in the same direction, separated by short reaches. Of bends of less than 45° central angle, the improved channel has only about one sixth as many as the unimproved channel. A summary of all data shows that in 1930 the medium-stage, unimproved channel had 147 bends, with a total angular curvature of 8,340°; whereas the future project channel is planned with 116 bends and a total angular curvature of 10,640°. At low stages in 1930, the thalweg had 269 bends with a total angular curvature of 14,286°.

The radii of curvature of the bends and general character of the meanderings of a river depend upon many factors, including the size of the stream, the slope of the valley, and the character of the sediment. The Mississippi River sweeps through its great curves in meanders of many thousands of feet radius, whereas some small stream flowing in curves of similar shape may have prevailing radii of only a few feet. On the section of the Missouri River considered in this study, experience has shown that, if otherwise properly designed, bends of 4,000 to 8,000 ft minimum radius are generally satisfactory and create no unusual hydraulic disturbance at any stages experienced in recent years. Straight reaches and bends with radii much greater than 8,000 ft have undesirable characteristics during low stages because of the excess number of crossings that may develop. These extra crossings introduce an extra sinuosity and impede the flow to that extent, besides being a hazard to navigation. On the other hand, bends of less than 4,000-ft radius, and particularly those of less than 3,000-ft radius, create excessive disturbance and turbulence in the river at bankful and overbank stages, and for this reason are no longer constructed. Fig. 6(*a*) shows that, of bends with central angles greater than 45°, the number with excessively sharp curvature has been greatly reduced by the channel improvement project. A separate study of bends of less than 45° central angle shows a similar result to have taken place, the bends of undesirable curvature in this class having been entirely eliminated by the improvement (see Fig 6(*b*)). If results for all bends in the river are combined, the number with less than any given radius is shown to have been decreased by the channel improvement.

It is believed that, considering solely the planimetric shape of the river, the final results of the improvement program will tend to increase the hydraulic efficiency of the river at all stages of bankful or less, due to reduction in sharp bends, irregular banks, excess crossings, and split channels. During overbank stages efficiency will tend to be somewhat less due to the greater curvature of the improved river, although promoted by the more regular shape, the reduction in the number of sharp bends, and other favorable factors. The relative importance of overbank stages as compared to lesser stages may be roughly estimated from the fact that from 1929 through 1937 (a subnormal period)

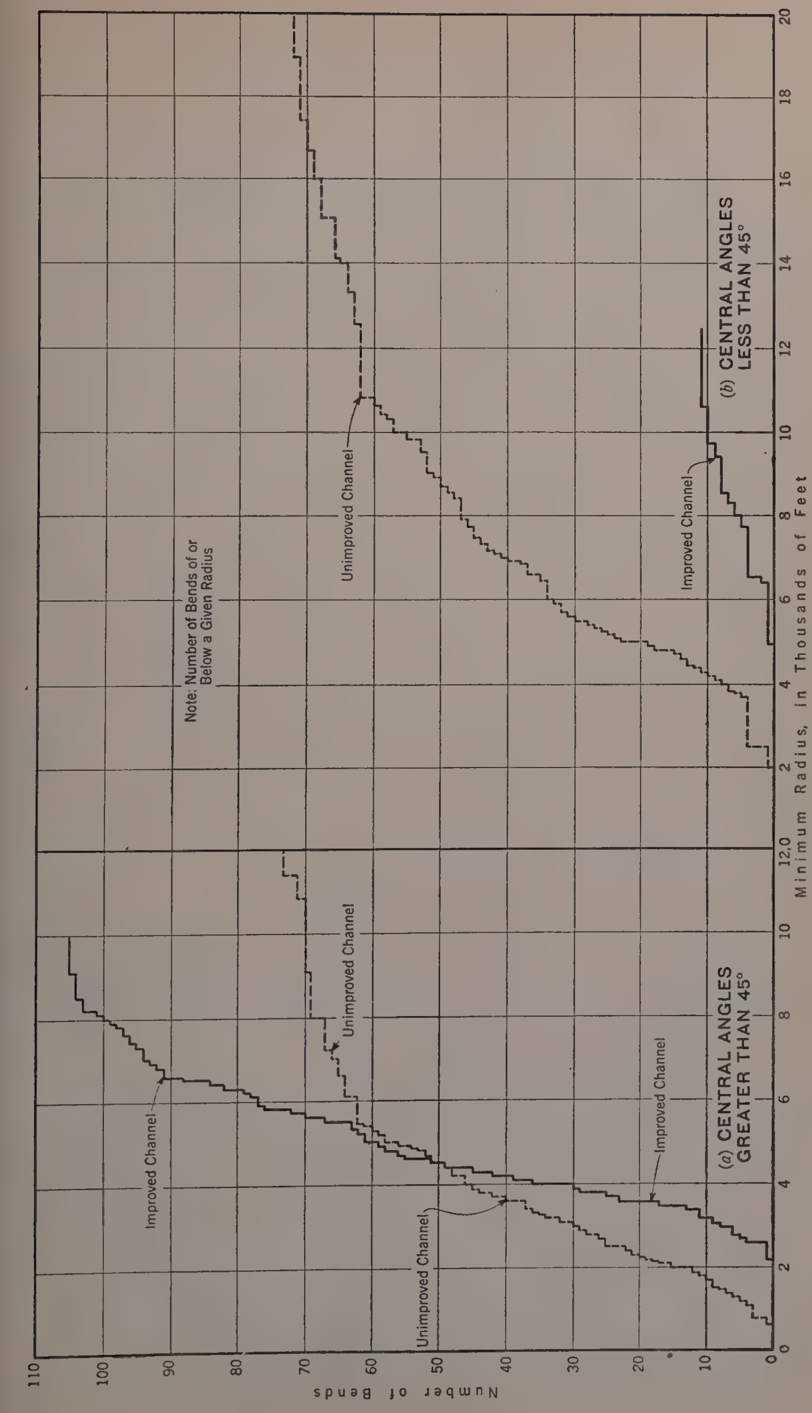


FIG. 6.—NUMBER OF RIVER BENDS CLASSIFIED BY THEIR MINIMUM RADII

the mean flow was about 23,000 cu ft per sec, whereas overbank flow does not start until flows exceed about 75,000 cu ft per sec, and is unimportant except near the crest of unusual floods.

*Fort Peck Reservoir.*—The regimen of the Missouri River will unquestionably be greatly influenced in some respects by the operation of the Fort Peck Reservoir. This reservoir, with a gross capacity of 19,400,000 acre-ft, controls the entire runoff of the Missouri River above the mouth of the Milk River. Its primary purpose is to regulate the flow of the Missouri River so as to insure required minimum flows during the navigation season at the head of the navigation improvement. Other benefits will result, due to partial flood control of the Missouri River, the production of incidental hydroelectric power, and possible utilization for irrigation.

In general, the effects of this reservoir will be as follows: During all years, flood peaks and overbank stages will be decreased; and during dry years, medium to overbank stages will probably also be reduced. A considerable part of the winter flow will also be stored. During August, September, October, and early November, there will be considerable releases, in practically all years, in order to bring the navigation project up to the desired minimum flow, and during wet years there may be additional excess releases during October in order to draw the reservoir down for flood-control storage. It seems probable that these changes will tend to prevent the aggradation of the channel which has been observed during dry cycles of the past, but in the long run this factor may be balanced by reduction in frequency of high discharges in wetter years. Although detailed studies have not been made for this paper, it is not believed that the operation of this reservoir alone will have an important effect, between Sioux City and Rulo, upon the average ratio between the quantity of sediment brought into the section and its sediment-carrying capacity, for a given slope.

*Irrigation Influences.*—The extensive past and projected future development of irrigation on a large scale in the Missouri Basin must be a factor tending toward aggradation of the stream bottom. Irrigation projects in the Missouri Basin are ordinarily installed where the water supply is comparatively free from sediment. In this manner a reduced supply of water is available downstream to carry the net sedimentary load of the stream, which is not decreased proportionately. No detailed study has been made as to the extent to which this effect would influence the slope of the Missouri River below Sioux City, but it is probably of minor importance.

*Velocity and Slope.*—Consideration has been given to approaching the problem of changes in velocity of the Missouri River due to the channel improvements by a comparison of velocities measured by the U. S. Geological Survey in connection with discharge readings over the period of record, from 1929 to the present. However, interpretation would present major difficulties, since the discharge readings were taken at points of the river contracted by bridges to approximately 1,000 ft, which widths have not materially changed since the general reduction in width of the river as a whole. Prior to the improvement work, the contracted sections near the gaging stations would naturally tend to have a slope and mean velocity lower than the average at low stages

and higher than the average at high stages of the river. The extent of these departures from the mean would be dependent upon the rate at which the river bed would scour out or silt in following changes in discharge. Data sufficient to determine the slope at these periods are almost entirely lacking. For these reasons, determinations of velocity changes, in order to be applicable to the river as a whole, can be computed by dividing the discharge at the nearest gaging station by the average area of the cross section as obtained by survey.

During 1938 a hydrographic survey and hydraulic study were made of a 25-mile section of river that was almost completely improved. Data were obtained at a discharge of 55,900 cu ft per sec with gages set at approximately 5-mile intervals. The part of the river considered included one long bend, of minimum radius 3,700 ft and central angle 161°; a sharp bend, of minimum radius 3,000 ft and central angle 104°; and three other bends of lesser severity. Results showed considerable variation in roughness coefficient. The mean slope of the entire section was 0.69 ft per mile; the average velocity 4.73 ft per sec; and the mean value of Manning's $n$, 0.0194.

As a check, a similar hydrographic survey and study were made for a 10-mile reach of almost wholly unimproved channel for the same year. This survey was taken at 95,000 cu ft per sec, which is slightly greater than overbank stage in this vicinity, and includes one fairly severe and one very abrupt bend. Only a very few improvement works had been constructed in the part of river under consideration. Values of roughness coefficient obtained varied in about the same range as the values obtained for the improved section of river. The mean slope was 0.91 ft per mile, the average velocity 4.80 ft per sec, and the mean value of Manning's $n$, 0.0197.

Overbank flow at these stages is of minor importance at this point. Any allowance for overbank flow would raise the roughness coefficient and lower the aforementioned average velocity, since these values were computed from the total discharge of the river at this time, as obtained from the nearest gaging station.

TABLE 1.—MISSOURI RIVER SLOPES

| Section (see Fig. 1) | 1890 milage reference | 1930 | | 1938 | | 1939 | 1940 |
|---|---|---|---|---|---|---|---|
| | | Length, in miles | Slope at low water, in feet per mile | Length, in miles | SLOPE, IN FEET PER MILE | | |
| | | | | | At low water | At flood | At low water |
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| Sioux City, Iowa...... | 804.2 to 799.2 | 43 | 1.06 | 5.3 | 0.87 | 0.99[a] | 0.93 |
| Blair, Nebr........... | 694.7 to 683.6 | 29 | 0.74 | 13.6 | 0.71 | 0.70 | 0.77 |
| Omaha, Nebr......... | 668.9 to 644.7 | 23 | 0.76 | 24.1 | 0.73 | 0.86[b] | 0.75 |
| Nebraska City, Nebr... | 607.8 to 601.9 | 20 | 1.25 | 5.7 | 0.83 | 1.26 | 0.88 |
| Brownville, Nebr...... | 586.0 to 570.8 | 14 | 0.90 | 12.3 | 0.77 | 0.88 | 0.93 |

[a] A 10-mile section. [b] A 17-mile section.

As of 1938, although construction was underway throughout most of the river between Sioux City and Rulo, there were only five parts of the river in which a single completed channel was established over a sufficient length for

comparative slopes to be available. These were, besides the section near Omaha already discussed, sections near Sioux City, and Blair, Nebraska City, and Brownville, Nebr. Recent slopes of these sections are given in Table 1.

It is noted in Table 1, Cols. 3 and 4, that the initial improvement by contraction of five stretches of the river resulted in all cases in a reduction of slope and a resultant slope below the average for the section between Sioux City and Rulo. The reduction was much greater for short sections and particularly the one section with a wide, divided, natural channel. Flood slopes in 1939 were greater on four of the five sections. Low-water slopes in 1940, although still below the general average, were all increased over 1938, coincidentally with the extension of contraction works to other parts of the river.

Values of roughness coefficient are apparently about the same for the improved and the unimproved section of the river. A computation by means of the Manning formula of the velocity that would obtain at each reach at a slope of one foot per mile shows that the unimproved section would have a velocity of 5.04 ft per sec, whereas the improved section would have a velocity of 5.71 ft per sec, although the data for the unimproved channel were taken at a 70% greater discharge. Similar comparisons made between present improved channels and unimproved sections of river in 1932 also show that the mean velocity obtaining, in terms of equal slope and discharge, is considerably higher for the improved river.

## Sediment Transporting Characteristics

*Data.*—In introducing this single problem the writer must acknowledge his indebtedness to comprehensive sediment studies made in this basin for the U. S. Engineer Department by L. G. Straub,[4] M. Am. Soc. C. E. For a general description of the sedimentary characteristics of this basin, as well as for a large amount of basic data, reference must be made to this work. For the section of river under consideration no data were taken between those of the 308 report[2] and a program that was undertaken early in 1939. In the 1939 program, bed-load and suspended sediment samples were taken at Omaha, near the center of the largest section of the improved river above Rulo, and at Yankton, S. Dak., which is many miles above the head of the navigation improvement and represents the unregulated river. The mean width of the river in the vicinity of Omaha is about 725 ft; at Yankton it is about 2,000 ft. The slope at Omaha is 0.71 ft per mile, and at Yankton it is 1.13 ft per mile, both slopes taken during October, 1938, over a stretch of river more than 8 miles long. For the period of sampling the discharge at Yankton was more than nine tenths of the Omaha discharge.

*Nature of Problem.*—The Missouri River carries a heavy bed load, which has been sampled and analyzed, but which has not been measured as to quantity. In order to establish a future equilibrium, the competence of the improved channel must be sufficient to move the largest particles brought down to it by the unimproved channel above; and, similarly, the capacity of the improved channel must be sufficient to move the total bed load of the unimproved channel. The Missouri River also carries a heavy suspended load, which has been both analyzed and measured. A third condition for equilibrium is that the

downstream part of the river must carry the entire suspended load brought to it by the upstream part.

If the river develops transporting characteristics over and above those sufficient to carry its total sediment load in a reach where its banks are stabilized, it will scour deeper, provided that the competence is sufficient to move the material of which the bed is composed.

*Suspended Sediment.*—Curves (1) to (3), Fig. 7, represent the average of mechanical analyses of suspended sediment at Yankton and Omaha. For 1939 a very close correspondence is shown between the sediment at Yankton

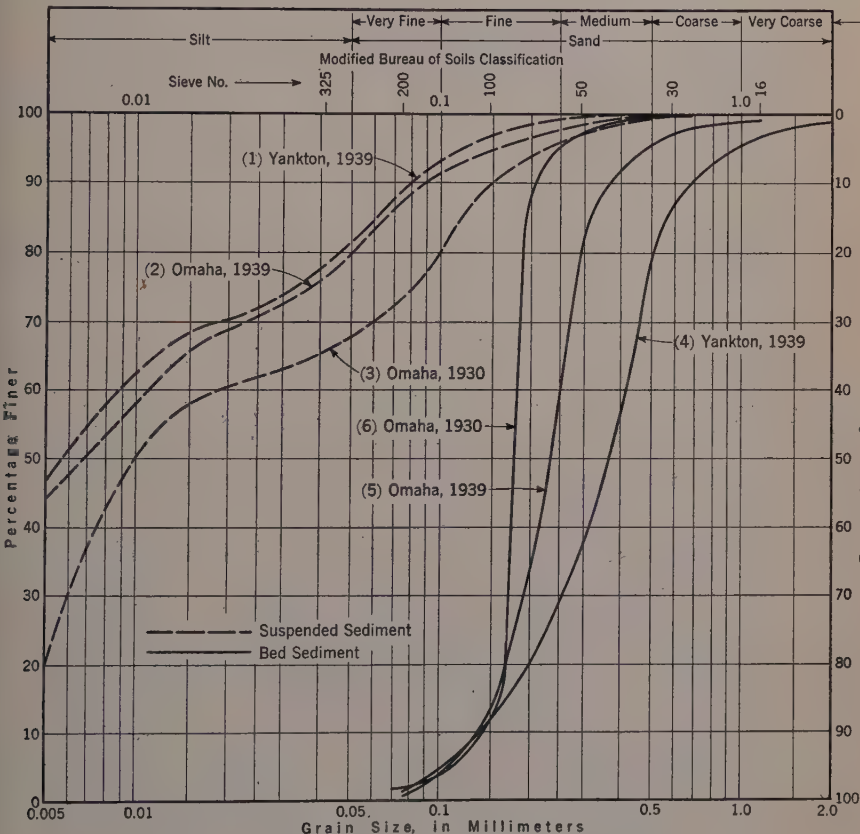


FIG. 7.—AVERAGE OF MECHANICAL ANALYSES

and Omaha, with the latter very slightly coarser. In these cases about 20% of the materials shown is very fine sand or coarser, as compared to about 32% as shown on the 1930 curve for Omaha. Fig. 8 shows the rating curves for the relationship between the discharge and the total suspended load at Yankton for 1939, and at Omaha for 1930 and 1939. The wide divergence between observed results at high discharges is noteworthy. In the case of the Missouri River the quantity of finer sediments carried is probably always dependent

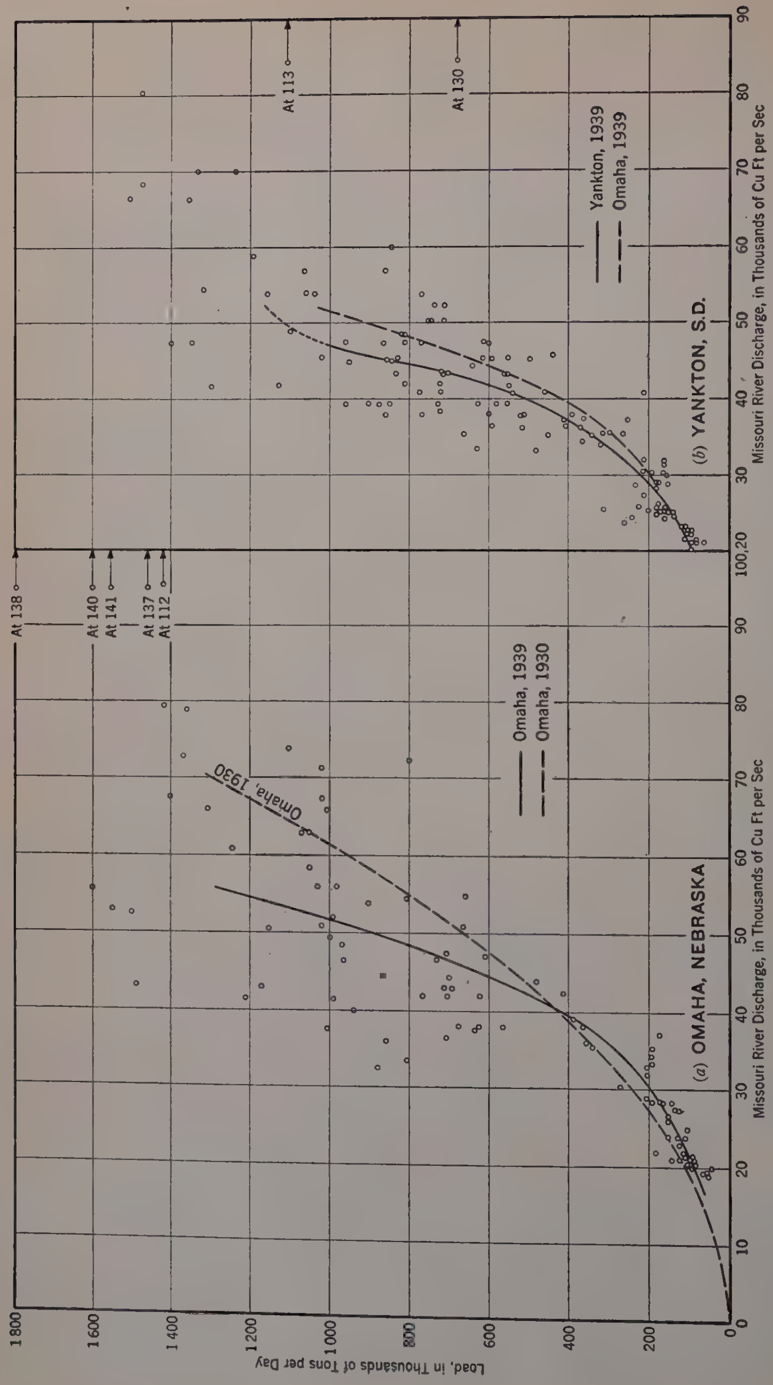


FIG. 8.—RATING CURVES FOR TOTAL SUSPENDED LOAD, 1939

upon the quantity that enters the channel rather than upon the capacity of the river under a given set of hydraulic conditions. The navigation works in progress above Omaha introduce artificial conditions of bank cutting, as well as of sedimentation, which tend to obscure any natural relationship that might otherwise have been found between ratios of finer suspended sediments at various points on the river. Therefore, these rating curves of total suspended load have little value except for calculation, by empirical means, of the total suspended load of the stream.

It has been known for some time, of course, that particles of grain size greater than about 1/16 mm act, when in suspension, under a law different from that which governs finer particles. It is also known that although clays and

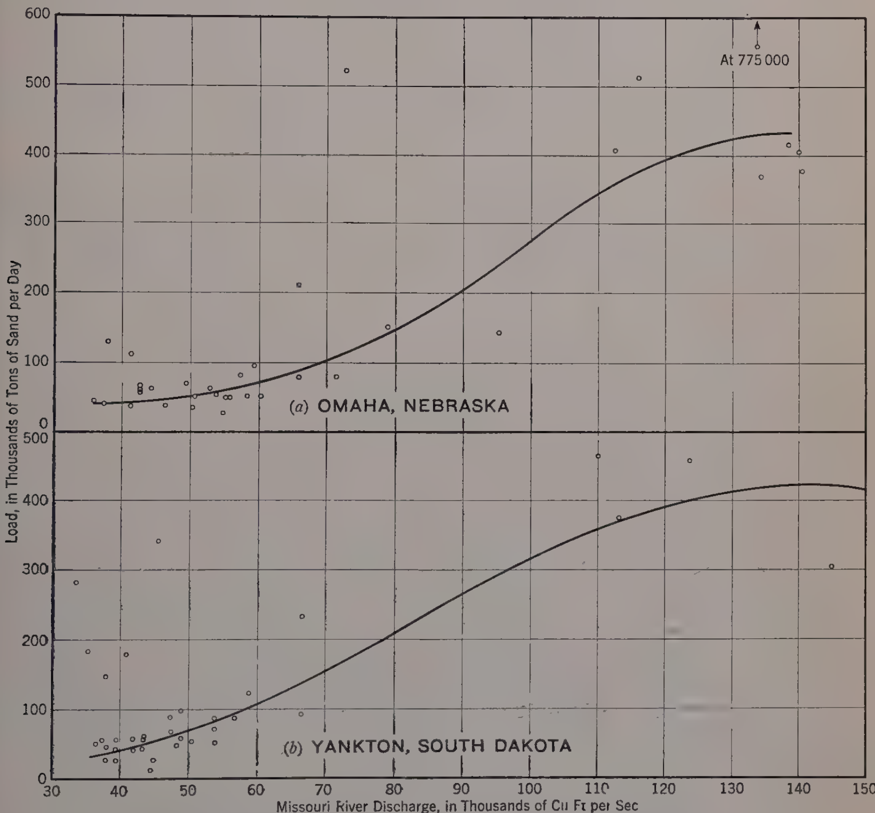


FIG. 9.—RATING CURVES FOR SAND IN SUSPENSION, 1939

silts, once compacted, resist erosion by velocities considerably in excess of that sufficient to carry them in suspension, sand particles are much more readily picked up, and are, moreover, present in much greater quantities in the bed of the Missouri River. With these facts in mind, curves were drawn representing the variation with discharge of the total volume of very fine sand (greater than 0.074 mm) and coarser materials in suspension in the river. These curves

are reproduced in Fig. 9. Unfortunately, they show that the suspended sand passing a given point, as well as the total suspended content, varies with parameters other than discharge. Variation in slope and velocity, due to the passage of sand waves on the bottom and to rising or falling stages, are probably among the influences to be considered. However, these local variations should balance when the average is taken of the considerable number of variations, and the mean quantity of suspended sand, rather than total suspended sediment, should be more nearly proportional to the capacity of the stream.

In order to make a more direct comparison of quantities of coarse suspended sediment (material coarser than 0.074 mm) at Omaha and Yankton, respectively, data were taken for a corresponding though non-consecutive period covering 29 days, from April through July, 1939, with allowance for a 2-day time of travel between Yankton and Omaha, in order that the data would be comparable. These are the only corresponding days on which composite samples were taken large enough to contain sufficient quantities of suspended sediment to permit mechanical analyses to be made. Results are given in Table 2.

TABLE 2.—Comparison of Data, April through July, 1939

| No. | Description | Omaha | Yankton | Ratio |
|---|---|---|---|---|
| 1 | Average water discharge, in cu ft per sec | 70,000 | 61,800 | 1.13 |
| 2 | Average coarse sediment discharge, in tons per day | 174,000 | 134,800 | 1.29 |
| 3 | Average total sediment discharge, in tons per day | 1,240,000 | 1,293,000 | 0.96 |

Results of Item 3, Table 2, are checked by the fact that for a period of 84 corresponding days (with allowance for travel time) the average total sedimentary discharge was exactly the same for the two stations.

From the foregoing it appears that the Missouri River at Omaha carried a 14% higher concentration of the coarser suspended sediments than it did at Yankton during the same period, the excess having been picked up by scour from the bed or banks or having been brought in by the tributaries. On the other hand, the content of finer sediments was reduced absolutely as well as relatively. Since silts and clays in any appreciable quantity do not settle in the channel of the river, there must have been a large deposit of the finer sediments on bars and behind dike systems between Yankton and Omaha. For lack of an available supply, the river is unable to replace this finer material by scour, although the capacity of the stream for the finer sediments is probably seldom if ever reached at any point, as shown by previous investigations by Professor Straub.

As far as suspended load is concerned, therefore, it appears that the improved section of the river has a tendency, at present, to pick up and carry a net addition to its suspended sediment content from its own bed and banks, even at slopes less than the average now prevailing. The conclusion appears justified that, for a given slope, the improved channel has a greater capacity than the natural river for the transportation of suspended sediment.

*Bed-Load Competence.*—Fig. 7(*b*) shows graphically the average mechanical analyses of bed sediment samples taken. The results show a predominance of fine and medium sand, with 1939 results for Omaha coarser than in 1930, but

not as coarse as those of Yankton in 1939. The Yankton results are probably influenced by its location below the mouth of the Niebrara River, which carries a heavy bed load, and the natural sorting processes present tend to create finer deposits in the lower reaches of a river than are found in the upper sections.

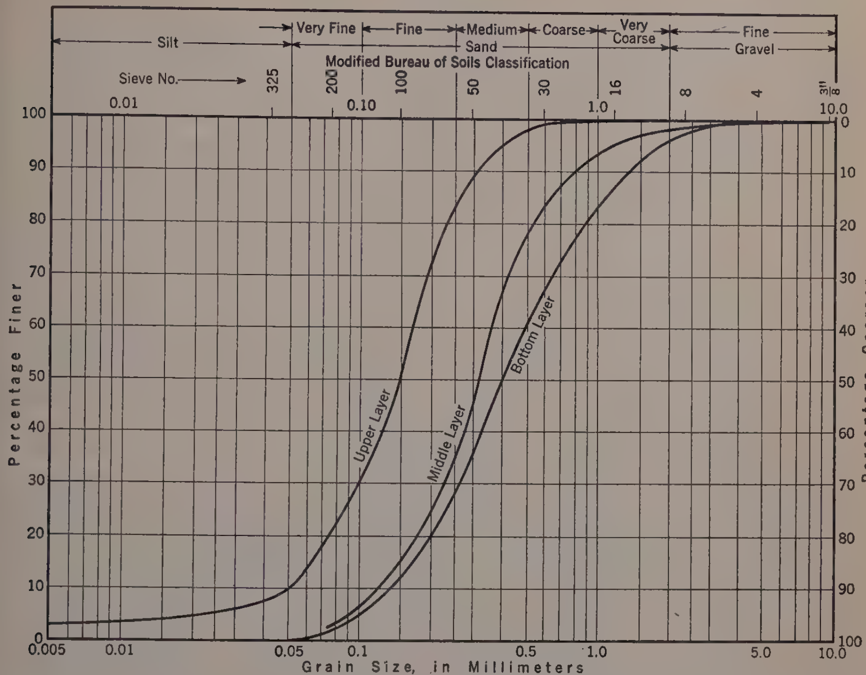


FIG. 10.—SOIL CLASSIFICATION, 10-FT LAYERS OF A BAR ISLAND, GAVINS POINT, YANKTON, S. DAK.

Fig. 10 shows results of analyses of approximately fifty samples taken from a bar island in the river a few miles above Yankton, separated into curves representing successive 10-ft layers from the surface. The lower surface of the bottom layer extends 5-ft below the deepest point of the stream bed at this locality. Results show that nearly all of this sediment is sand, with only 4% of the bottom layer exceeding 2 mm in diameter. Other analyses and field inspections at hundreds of other points throughout the length of the river confirm this general result: There is little gravel in the bed of the Missouri River between Yankton and Rulo except near the mouth of the Platte River.

W. W. Rubey[5] indicates that competence is governed by tractive force formulas rather than the sixth power law when the particles are equal to or finer than a mean diameter of 0.688 mm. Fig. 7 (curves (4), (5), and (6)) and Fig. 10 indicate that Missouri River bed loads may be expected to fall within this range, and therefore should be governed by tractive force formulas.

As has been shown previously, the available slope in the Missouri River in the part being considered is 0.96 ft per mile. It is necessarily assumed that

[5] "The Force Required to Move Particles on a Stream Bed," by W. W. Rubey, *Professional Paper 189-E*, U. S. Geological Survey, 1937.

this was sufficient to carry the average bed load with the prevailing discharges prior to the improvement. Roughness coefficients and discharge schedules have been little changed, but mean width is being reduced from 3,650 ft to 725 ft.

According to the tractive force criterion, competence equal to that which prevailed previously will be retained if the depth-slope product remains constant. Applying this and other conditions, given previously, to the Manning formula, a slope of 0.24 ft per mile would be sufficient to retain competence unchanged.

A similar result is obtained by applying an equation derived from the Schoklitsch formula[6] by Samuel Shulits, Assoc. M. Am. Soc. C. E., and W. E. Corfitzen, Jun. Am. Soc. C. E.:

$$D_{max} = 4{,}770 \frac{Q\, S^{4/3}}{B} \qquad (1)$$

in which $D_{max}$ is the diameter of the largest grain size moved in millimeters; $Q$ is the discharge in cubic feet per second; $S$ is the energy-gradient; and $B$ is the width of the river in feet. According to Eq. 1 a slope of 0.285 ft per mile would be sufficient to maintain competence unchanged.

This result is particularly interesting inasmuch as Messrs. Shulits and Corfitzen were attempting to define not only competence but also, apparently, conditions of a non-silting, non-scouring channel at an actual canal location; and they considered the results from this formula to fall within the approved band of results. To quote, "Hence, the maximum particle-size that can be transported as part of the bed load represents the size of material which would create a non-silting, non-scouring channel." According to this interpretation a slope of 0.285 ft per mile would not only provide competence equal to that existing previously, but would also serve, apparently without further analysis, as the equilibrium slope of the contracted channel.

Messrs. Shulits and Corfitzen also quote a table of critical tractive forces observed by the Nuernberg Kulturamt.[7] For ordinary quartz sand from 0.4 to 1.0 mm in diameter the critical tractive force required was 0.25 to 0.30 kg per sq m. Checking these data by ordinary measurement is complicated by the fact that, when taking a bed-load sample, it is impossible to determine whether or not this sample is in process of movement at the time. Therefore, of the various bed sediment samples for Omaha in 1939 (which were taken at discharges averaging 46,000 cu ft per sec) it is not known whether all samples were taken from moving deposits. However, the assumption will be made that the average sample would move at the average discharge. Substituting the known hydraulic characteristics of the river in the vicinity of Omaha into the Manning formula an average depth of 12.9 ft is obtained for discharges of 46,000 cu ft per sec and the present average slope of 0.71 ft per mile for the nine miles in the vicinity of the gaging station. Substituting, a tractive force of 0.53 kg per sq m is obtained as compared to the 0.25 to 0.30 of the Kulturamt results.

[6] "Bed-Load Transportation and the Stable-Channel Proper," by S. Shulits and W. E. Corfitzen, *Transactions*, Am. Geophysical Union, 1937, p. 456.

[7] "Der Wasserbau," by A. Schoklitsch, Julius Springer, Vienna, Vol. 1, 1930, p. 127.

Examining the average mechanical analysis of bed-sediment samples, at Omaha (Fig. 7), only 1% of the sample was found to be coarser than 1.0 mm. It is probable that this small percentage of larger particles was either moved, during the discharges, much higher than the average that occurred during the period considered, or consisted of particles of shape and specific gravity considerably different from those of ordinary sand particles. For the average discharge it is believed that 1.0 mm represents, as nearly as can be estimated, the largest particle that was moving. Moreover, when mean grain diameter is plotted against discharge on the day the sample was taken, there is comparatively little variation. Mean grain diameter increased less than 50% between 20,000 and 140,000 cu ft per sec, and the three coarsest samples were actually taken at fairly low discharges. The only reasonable explanation for these results is that the Missouri River does not now find, in its bed, sediments as coarse as those which it is competent to move at fairly high discharges. This being the case, the criteria of competence cannot define the future equilibrium of the channel except possibly below the mouths of gravel bearing tributaries.

*Capacity.*—The Schoklitsch bed-load formula is:

$$G = \frac{86.7\, S^{1.5}}{\sqrt{D}} (Q - B\, q_0) \qquad (2)$$

in which $G$ is the bed load in pounds per second; and $D$ is the grain-diameter in inches. In this formula the factor $B\, q_0$ represents the discharge at which bed-load movement begins, which has been shown in the Missouri River 308 report to be very small in this case, in proportion to the average discharge. Therefore, since the discharge schedule will have no changes of material effect, changes in the expression $Q - B\, q_0$, as between the improved and unimproved channel, can be dismissed as negligible. Surprizingly enough, there are no other factors in this equation that show any variation under the conditions stated. Therefore, Eq. 2 must involve some inherent assumption as to the relationship between width and slope, or width and discharge, which entirely unfits it for present uses. Since this equation was principally based on flume experiments, its empirical form renders its application to natural streams questionable unless verification can be made on a large scale.

Formulas probably more applicable to the conditions in the Missouri River were developed by Professor Straub[8] for the 308 report. His applicable equation is:

$$G_w = \frac{u\, i^{1.4}\, Q^{3/5}}{c^{1/2}} (Q^{3/5} - Q_0^{3/5}) \qquad (3)$$

in which $G_w$ is the quantity of sediment transported along the stream bed in pounds per unit width of channel; $Q_0$ is the discharge per unit width of channel (for a slope $i$) at which sediment transportation begins, $c$ is the roughness coefficient in an open-channel formula, corresponding to the Manning expression $\frac{1.486}{n}$, and $u$ is the "sediment characteristic," depending upon the physical

[8] "Effects of Channel-Contraction Works Upon Regimen of Movable Bed Streams," by L. G. Straub, *Transactions*, Am. Geophysical Union, 1934.

characteristics of the sediment carried. For present purposes $Q_0$ may be neglected, since, although it may be varied somewhat by the contraction of the channel, it was very small (even prior to contraction) in proportion to the average discharge. The quantity of sediment, $G_w$, must be assumed to be the same for the improved or unimproved channel, which is also true of $Q$ and the sediment characteristic $u$. Substituting known characteristics, the reduction in width of the channel from 3,650 to 725 ft is found to cause a reduction in the equilibrium slope from 0.96 to 0.76 ft per mile, as an equilibrium condition for a bed load for which the steeper slope would otherwise have been an equilibrium condition. This accords quite closely with the slope of 0.69 ft per mile which is actually in existence in the 25-mile improved stretch in the vicinity of Omaha at the present time, and with slopes on other improved parts of the Missouri River.

In fairness to Professor Straub it must be noted that his formula was designed for a uniform discharge, whereas the Missouri River discharge is variable; and that he contemplated measurement of widths at low water, whereas the data used are taken for convenience at medium stages. Greater refinements would probably show slightly different results from the foregoing. Investigations such as those summarized in this paper bring to light great deficiencies in basic data as to the principal hydraulic characteristics of major streams. A few more years of study may provide data which will change, materially, the incomplete and short-period records utilized for this paper.

Using all data available, a very approximate estimate could be made as to the time required for a 250-mile stretch of river with an existing slope of 0.96 ft and an equilibrium slope of 0.76 ft to scour its bed by any given amount. This attempt will not be made herein because, although certain parts of the river are practically completed, there are other parts where the channel is only partly formed, and where contracted sections, split channels, and protruding dikes, not yet silted in, form obstructions to flow that interfere with, and delay, the process and would render any estimate in quantitative terms very approximate. However, recent stage discharge relationships show that the process is already under way.

## Conclusions

It is concluded that locally steep slopes are caused by rock protrusions, sharp bends, crossings, tributary influences, and reentrant chutes, besides temporary factors. Long meanders also increase slope at overbank stages. Since the improvement project will reduce the number of crossings, eliminate chutes, and reduce the number of sharp bends, its general shape will tend to allow the river to reach a flatter slope, even though at overbank stages the additional curvature of the improved river is a counteracting influence.

It is concluded that Fort Peck Reservoir will probably not have any important effect on the mean slope of the stream; and that future irrigation development, although tending toward aggradation, will probably be of slight importance.

The improved and unimproved channels have equal roughness coefficients in the reaches examined; and the improved channel has a greater mean velocity relative to the slope, due to its greater hydraulic radius. Improved sections of the river now maintain themselves at less than the present (or the 1930) average slope.

It is concluded that during 1939 the Missouri River in the improved section at Omaha had a greater capacity for suspended sediment than it had on the unimproved section at Yankton, despite a considerably flatter slope.

An analysis of comparative bed-load competence of the improved and unimproved rivers shows that the tractive force criterion, and an equation derived from the Schoklitsch formula by Messrs. Shulits and Corfitzen, both indicate that a future equilibrium slope of less than 0.3 ft per mile would be sufficient to retain competence unchanged in the contracted channel. Since the Missouri River does not now find in its bed sediments as coarse as those which it is competent to carry at high discharges, it is concluded that bed scour of this stream is limited by capacity, and that formulas for competence will not define an equilibrium condition in this case. The Schoklitsch bed-load formula is inapplicable to the change of regimen of the Missouri River.

Applications of Professor Straub's bed-load formula indicate that, if other things remain equal, contraction of the Missouri River should increase its capacity for bed load by 38%, or reduce from 0.96 to 0.76 ft per mile the average slope at which a section could carry an unchanged bed load. Slopes on parts of the river first contracted correspond fairly well with this analysis. In view of the great length of river involved, no appreciable change in mean slope can be expected; but a progressive lowering of the bed of the stream through scour will probably continue for some time.

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PAPERS

# ANALYSIS OF BUILDING FRAMES WITH SEMI-RIGID CONNECTIONS

By Bruce Johnston,[1] Assoc. M. Am. Soc. C. E., and
Edward H. Mount,[2] Esq.

## Synopsis

Methods applicable to the analysis of building frames with semi-rigid riveted or welded connections between the beams and columns are presented in this paper. The methods of analysis are too complex for ordinary design use, but the writers have presented simple design procedures, based on these methods of analysis, elsewhere (2) (3),[3] and have made them expeditious by the use of charts and diagrams. Such design methods effect permissible economy in the required beam sizes, made possible by considering the partial restraint afforded by standard or near standard connections, particularly riveted or welded connections of the top and seat angle type.

This paper also presents test results of a welded building frame that corroborate the methods of analysis. A study of the effect of neglecting the width of members in the analysis is presented. ("Member width" is used in this paper to indicate column width or beam depth, as the frame is viewed in elevation.) The essential features of the methods of analysis have been presented elsewhere (1) (13) (14), and it is the intention of the writers to modify them slightly so as to simplify the technique by conforming in every way to the usual slope-deflection and moment-distribution procedures.

## Introduction

The design of the steel frames that form the skeleton of multiple-story steel buildings is usually based upon certain simplifying assumptions, chief of which are: (*a*) For the purpose of beam design the beam-column connections

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **July 15, 1941.**

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[3] Numerals in parentheses, thus: (2) (3), refer to corresponding numbers in the Bibliography; see Appendix.

are assumed to be pin connections, or simple supports; (*b*) columns are designed without attempting to evaluate the moments introduced by frame action; and (*c*) the beam-column connections are assumed to be rigid in calculating stresses due to lateral or wind loads.

These assumptions have afforded a means of rapid design calculation. Riveted building frames constructed on the basis of these design assumptions have proved to be safe and reliable, but there remains the possibility of achieving greater economy through the use of more nearly correct design assumptions. According to the British investigations (1), an average saving in the weight of beams of as much as 20% may be expected by taking advantage of the partial end restraint of riveted beam-column connections. Welded construction, with its inherent continuity, also makes similar saving in weight possible.

The basis for the application of more accurate design methods to frames with semi-rigid connections has already been laid in Great Britain (1) and in work of a parallel nature in the United States (2) (3) (4). The comparison of analytical and experimental results presented in this paper was made possible by the construction and test of a welded building frame with three bays and two stories. The beam-column connections used in this frame were of a semi-rigid type previously studied at the Fritz Engineering Laboratory (2) (5) in connection with research programs sponsored by the American Welding Society. Similar tests on riveted building frames have been made in Great Britain (1).

The slope-deflection and moment-distribution methods have as a common purpose the determination of the bending moments at the ends of the individual members of a statically indeterminate frame. Both methods in their usual form are based on the assumption that the deformations of the frames are caused entirely by bending of the members and that the relation between bending moment and distortion is given by the beam formula:

$$\frac{M}{E\,I} = \frac{d^2y}{dx^2} \qquad (1)$$

The derivation of the beam theory and the assumptions on which it is based may be found in any text on the strength of materials.

In 1915 the slope-deflection method was applied in the United States (by Wilbur M. Wilson and George A. Maney, Members, Am. Soc. C. E.) to the analysis of wind stresses in tall buildings (6). A more complete treatment followed in 1918 (by Professor Wilson, with F. E. Richart and Camillo Weiss, Members, Am. Soc. C. E.) (7), and in 1931 a modification (8) was introduced by L. T. Evans, Assoc. M. Am. Soc. C. E., to take care of members with varying moments of inertia. The method of moment distribution was first presented in mimeographed form by Hardy Cross, M. Am. Soc. C. E., in 1926 (9) (10) (12). Innumerable variations and short cuts have been applied to the moment-distribution method and, although some of these have merit, the original method remains the outstanding development in recent times for rapid and effective analysis of continuous frames. The application of both the slope-deflection and moment-distribution methods to the analysis of frames with semi-rigid connections was made by John F. Baker, Assoc. M.

Am. Soc. C. E. (13). The width of the members and the semi-rigid nature of the connections are both taken into account, as it is found that the neglect of member width gives rise to considerable error, particularly in the case of analyses of frames with semi-rigid connections.

## Analysis of Frames with Semi-Rigid Joints

It will be assumed that the reader is already familiar with the usual application of the slope-deflection and moment-distribution methods, for which references are readily available (12) (15) (16) (17). The methods herein presented are identical in mode of application to the usual simple form, with the exception that special coefficients must be used in the slope-deflection equations and for the carry-over and distribution factors in the moment-distribution method. The following method is identical with that previously presented by Professor Baker (13) (14) when the width of member is neglected. When the width of member is considered, the following method differs in two respects from that of Professor Baker: (*a*) The interior of the joint between connections is assumed infinitely rigid, whereas Professor Baker assumes it to have the same stiffness as the beam; and (*b*) hypothetical moments are computed at the joint centers by the usual slope-deflection and moment-distribution procedures, whereas in Professor Baker's method separate expressions are given for the moment and shear at the connection, or column face, and are dealt with separately.

The following assumptions apply both to the slope-deflection and to the moment-distribution relations as herein presented: (*a*) Members are of uniform cross section between their end connections; (*b*) the semi-rigid connection at the end of a member behaves elastically as defined by the connection constant $\gamma$; and (*c*) the interior of the joint between connections is assumed to be infinitely rigid, although free to rotate as a rigid body.

The notation used is shown subsequently in Figs. 5 and 6. The hypothetical moments and shears at the joint centers, when width of column is considered, are designated by the bar above the letter $M$ or $V$, thus; $\bar{M}$ and $\bar{V}$.

*The "Semi-Rigid" Connection.*—The semi-rigid connection may be thought of as a locally weakened section between the end of the beam and the face of the column to which the connection is made. The effect on analysis is the inverse of the effect produced by end haunches or added cover plates. The typical test behavior of a riveted or welded connection of this type is shown in Fig. 1, which presents the plot of the relationship between moment transmitted through the connection and the angle change between the joint center and the end of the beam. In the design range the relationship is assumed linear and the inverse slope is termed the connection constant, $\gamma$, thus:

$$\gamma = \frac{\phi}{M} = \frac{\text{Angle change}}{\text{Moment}} \quad \text{.....................} \quad (2)$$

The connection constant $\gamma$ may be defined as "angle change for unit moment" and may be determined experimentally by testing typical joints. The vertical

line through the origin in Fig. 1 would indicate the behavior of a perfectly fixed connection with $\gamma = 0$, and the horizontal line would represent the behavior of a frictionless pin connection, in which case $\gamma = \infty$. Fig. 2(*a*)

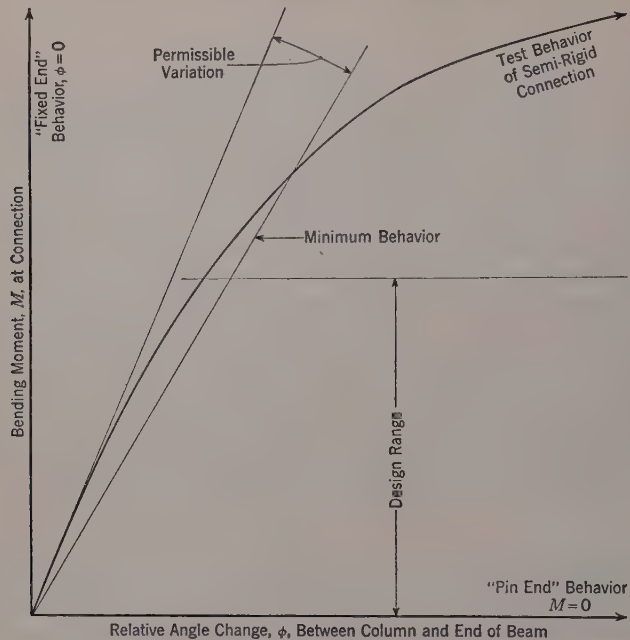



FIG. 1.—TYPICAL $M$-$\phi$ RELATION IN THE TEST OF A WELDED OR RIVETED BEAM-COLUMN CONNECTION

shows the test setup for determining the connection constant at an interior joint of a frame with beam-to-column-flange connections, and Fig. 2(*b*) shows a similar setup to test the connection between a beam and an exterior column

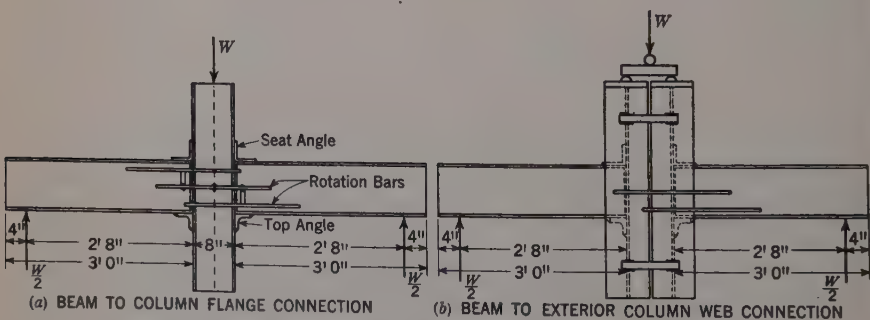



FIG. 2.—TEST ARRANGEMENT TO DETERMINE CONNECTION CONSTANTS

web. These connections correspond to those used on two frames tested by the writers. The relative rotation between the ends of the beam and the center of the column at the joint were measured with a 20-in. level bar which is shown in Fig. 3 in position for measurement of angle changes of the actual

test frame. Fig. 4 is a graph within the test-design range of measured angle change plotted against moment in typical joint tests corresponding to the actual test frame.

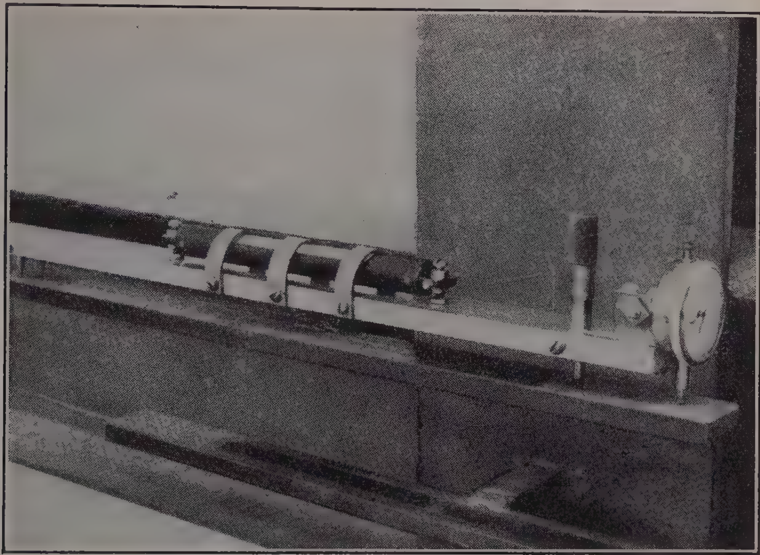


FIG. 3.—LEVEL BAR USED TO MEASURE ROTATIONS

*The Slope-Deflection Equations.*—For any individual member of a frame, the relation between its end moments, the angle changes at each end, and the angle change of the member as a whole may be expressed by a pair of slope-deflection equations. For the usually assumed case of uniform beam cross

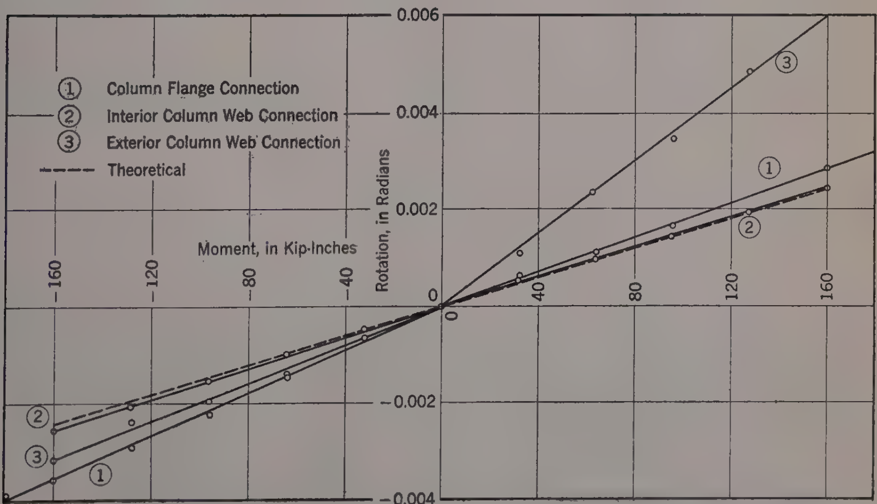



FIG. 4.—TEST RESULTS IN DESIGN RANGE FOR TYPICAL BEAM-COLUMN CONNECTIONS

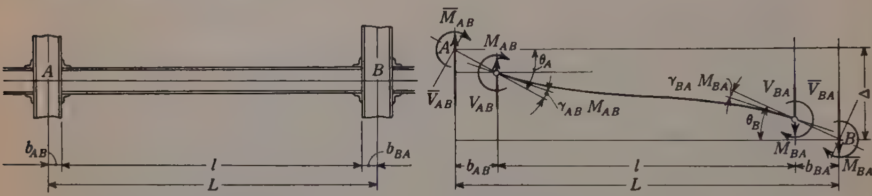
section, and with rigid end connections, these equations are written:

$$M_{AB} = 2\,E\,K\,(2\,\theta_A + \theta_B - 3\,R) - M_{RAB} \qquad (3a)$$

and

$$M_{BA} = 2\,E\,K\,(2\,\theta_B + \theta_A - 3\,R) + M_{RBA} \qquad (3b)$$

In Eqs. 3, $K = \frac{I}{l}$, in which $I$ = the moment of inertia and $l$ = the length of the member (distance between joint connections). The angle changes at ends $A$ and $B$ are $\theta_A$ and $\theta_B$; and $R = \frac{\Delta}{l}$ = the angle change of the entire member, $\Delta$ being the relative lateral displacements of ends $A$ and $B$. The fixed-end moments for the lateral loads alone are $M_{RAB}$ and $M_{RBA}$. The slope-deflection equations may be derived directly from Eq. 1 or by an application of the moment-area principles.




(a) SEMI-RIGID CONNECTIONS AND FINITE WIDTH OF MEMBER

(b) DEFORMATION OF MEMBER WITH SEMI-RIGID CONNECTIONS AND FINITE JOINT WIDTH

Fig. 5.—Deformation of Member Considering Semi-Rigid Connections and Finite Joint Width

Slope-deflection equations such as Eqs. 3 may be derived by similar methods for members which frame with semi-rigid connections. Fig. 5 shows the notation used and the geometry of the general deflected curve of any such member. Note especially Fig. 5(*b*), which shows the hypothetical moments at the center of the joint. The slope-deflection equations corresponding to Eqs. 3 for the hypothetical moments $\bar{M}_{AB}$ and $\bar{M}_{BA}$ at the joint centers for any member $A\,B$, as shown in Fig. 5, may be written:

$$\bar{M}_{AB} = \frac{1}{1 + 2\,\alpha + 2\,\beta + 3\,\alpha\,\beta}\,[2\,E\,K(C_{AA}\,\theta_A + C_{AB}\,\theta_B - C_{AC}\,R) - F_{AA}\,M_{RAB} - F_{AB}\,M_{RBA}] - V_{AB}'\,b_{AB} \qquad (4a)$$

and

$$\bar{M}_{BA} = \frac{1}{1 + 2\,\alpha + 2\,\beta + 3\,\alpha\,\beta}\,[2\,E\,K(C_{BB}\,\theta_B + C_{BA}\,\theta_A - C_{BC}\,R) + F_{BB}\,M_{RBA} + F_{BA}\,M_{RAB}] + V_{BA}'\,b_{BA} \qquad (4b)$$

Except for the fact that new coefficients replace the even integer coefficients in Eqs. 3, the application of Eqs. 4 to any particular problem is identical with the usual slope-deflection procedure.

In Eqs. 4 the new constants $C_{AA}$, $C_{AB}$, $C_{AC}$, $C_{BB}$, $C_{BA}$, $F_{AA}$, $F_{AB}$, $F_{BA}$, and $F_{BB}$ depend on the dimensions of the members and on the value of the joint constant. Factor $K$ again is given by $\frac{I}{l}$, and it should be noted that $l$ is the length between connections rather than the length between joint centers.

The connection constants $\gamma_A$ and $\gamma_B$ for the connections at the two ends of the beam are introduced into new constants $\alpha$ and $\beta$ by the relations (1):

$$\alpha = 2\,E\,K\,\gamma_A \qquad (5a)$$

and

$$\beta = 2\,E\,K\,\gamma_B \qquad (5b)$$

The fixed-end moments for a member with fixed, rigidly connected, ends of span length $l$ are $M_{RAB}$ and $M_{RBA}$; and $V_{AB}'$ and $V_{BA}'$ are the shears or reactions at the ends of a member with freely supported ends and span length $l$.

The constants $C_{AA}$, $C_{AB}$, $C_{AC}$, $F_{AA}$, and $F_{AB}$ in Eq. 4*a* are given in Table 1 for four different cases. All four cases are for unsymmetrical conditions, the first and the third considering semi-rigid connections, and the second and fourth considering rigid connections. In cases I and II, a finite width of member is considered; and, in cases III and IV, width is ignored. The values in case IV are those commonly assumed—that is, frames with rigid joints and with width of member neglected. In Eq. 4*b* the constants $C_{BB}$, $C_{BA}$, $C_{BC}$, $F_{BB}$, and $F_{BA}$ for $\bar{M}_{BA}$ are obtained from $C_{AA}$, $C_{AB}$, $C_{AC}$, $F_{AA}$, and $F_{AB}$, respectively, by interchanging $\alpha$ and $\beta$ and the subscripts $A$ and $B$. It is noted that $C_{BA}$ is equal to $C_{AB}$.

Eqs. 4 with the preceding coefficients are for moments at the joint centers and therefore are used with exactly the same equilibrium conditions as in the simpler form (2); namely,

For joint equilibrium,

$$\Sigma \bar{M} = 0 \qquad (6a)$$

and, for story equilibrium,

$$\Sigma \bar{M} + V\,h = 0 \qquad (6b)$$

In Eq. 6*b*, $h$ is the story height between neutral axes of two beams.

With Eqs. 4 it is now possible to determine the hypothetical end moments $\bar{M}_{AB}$ and $\bar{M}_{BA}$ at the joint centers. Moments and shears are assumed as positive when they act on the ends of the beam with a clockwise sense, or act on the joint with a counterclockwise sense. The hypothetical shears in the joint are constant and equal to the actual shear at the connection. The shears $\bar{V}_{AB}$ and $\bar{V}_{BA}$ may be calculated from the moments $\bar{M}_{AB}$ and $\bar{M}_{BA}$ by the following:

$$V_{AB} = \bar{V}_{AB} = -\left(\frac{\bar{M}_{AB} + \bar{M}_{BA}}{L}\right) + \bar{V}_{AB}' \qquad (7a)$$

and

$$V_{BA} = \bar{V}_{BA} = -\left(\frac{\bar{M}_{AB} + \bar{M}_{BA}}{L}\right) - \bar{V}_{BA}' \qquad (7b)$$

in which $\bar{V}_{AB}'$ and $\bar{V}_{BA}'$ are the end shears in a member having span length $L$ with simply or freely supported ends. The moments at the connections, $M_{AB}$ and $M_{BA}$, may now be calculated from the following:

$$M_{AB} = \bar{M}_{AB} + \bar{V}_{AB}\,b_{AB} \qquad (8a)$$

and

$$M_{BA} = \bar{M}_{BA} + \bar{V}_{BA}\,b_{BA} \qquad (8b)$$

## TABLE 1.—Slope-Deflection and Moment-Distribution Constants—Nonsymmetrical Cases

| Constant | Case I—Nonsymmetrical, semi-rigid connections, finite width of members | Case II—Nonsymmetrical, rigid connections, finite widths | Case III—Nonsymmetrical, semi-rigid connections, zero width | Case IV—Rigid connections, 0 width |
|---|---|---|---|---|
| | *(a)* Slope Deflection | | | |
| $C_{AA}$ | $2+3\beta+6(1+\beta)\frac{b_{AB}}{l}+3(2+\alpha+\beta)\frac{b_{AB}^2}{l^2}$ | $2+\frac{6\,b_{AB}}{l}+\frac{6\,b_{AB}^2}{l^2}$ | $2+3\beta$ | 2 |
| $C_{AB}=C_{BA}$ | $1+3(1+\alpha)\frac{b_{AB}}{l}+3(1+\beta)\frac{b_{BA}}{l}+3(2+\alpha+\beta)\frac{b_{AB}\,b_{BA}}{l^2}$ | $1+\frac{3\,b_{AB}}{l}+\frac{3\,b_{BA}}{l}+\frac{6\,b_{AB}\,b_{BA}}{l^2}$ | 1 | 1 |
| $C_{BB}$ | $2+3\alpha+6(1+\alpha)\frac{b_{BA}}{l}+3(2+\alpha+\beta)\frac{b_{BA}^2}{l^2}$ | $2+\frac{6\,b_{BA}}{l}+\frac{6\,b_{BA}^2}{l^2}$ | $2+3\alpha$ | 2 |
| $C_{AC}$ | $3(1+\beta)+3(2+\alpha+\beta)\frac{b_{AB}}{l}$ | $3+\frac{6\,b_{AB}}{l}$ | $3(1+\beta)$ | 3 |
| $C_{BC}$ | $3(1+\alpha)+3(2+\alpha+\beta)\frac{b_{BA}}{l}$ | $3+\frac{6\,b_{BA}}{l}$ | $3(1+\alpha)$ | 3 |
| $F_{AA}$ | $1+2\beta+(1-\alpha+2\beta)\frac{b_{AB}}{l}$ | $1+\frac{b_{AB}}{l}$ | $1+2\beta$ | 1 |
| $F_{AB}$ | $\beta+(\beta-2\alpha-1)\frac{b_{AB}}{l}$ | $-\frac{b_{AB}}{l}$ | $\beta$ | 0 |
| $F_{BB}$ | $1+2\alpha+(1-\beta+2\alpha)\frac{b_{BA}}{l}$ | $1+\frac{b_{BA}}{l}$ | $1+2\alpha$ | 1 |
| $F_{BA}$ | $\alpha+(\alpha-2\beta-1)\frac{b_{BA}}{l}$ | $-\frac{b_{BA}}{l}$ | $\alpha$ | 0 |
| | *(b)* Moment Distribution | | | |
| $\bar{M}_{SAB}$ semi-fixed end moment at joint center | $\frac{\left\{\left[1+2\beta+\frac{b_{AB}}{l}(1+2\beta-\alpha)\right]M_{RAB}+\left[\beta-\frac{b_{AB}}{l}(1+2\alpha-\beta)\right]M_{RBA}\right\}}{1+2\alpha+2\beta+3\alpha\beta}+V_{AB}'\,b_{AB}$ | $\left(1+\frac{b_{AB}}{l}\right)M_{RAB}-\frac{b_{AB}}{l}M_{RBA}+V_{AB}'\,b_{AB}$ | $\frac{[(1+2\beta)M_{RAB}+\beta M_{RBA}]}{1+2\alpha+2\beta+3\alpha\beta}$ | $M_{RAB}$ |
| $r_{AB}$ carry-over factor between joint centers | $\frac{1+3(1+\alpha)\frac{b_{AB}}{l}+3(1+\beta)\frac{b_{BA}}{l}+3(2+\alpha+\beta)\frac{b_{AB}\,b_{BA}}{l^2}}{2+3\beta+6(1+\beta)\frac{b_{AB}}{l}+3(2+\alpha+\beta)\frac{b_{AB}^2}{l^2}}$ | $\frac{1+\frac{3\,b_{AB}}{l}+\frac{3\,b_{BA}}{l}+\frac{6\,b_{AB}\,b_{BA}}{l^2}}{2+\frac{6\,b_{AB}}{l}+\frac{6\,b_{AB}^2}{l^2}}$ | $\frac{1}{2+3\beta}$ | $\frac{1}{2}$ |
| $\bar{S}_{MAB}$ end rotation stiffness | $\frac{2EK\left[2+3\beta+6(1+\beta)\frac{b_{AB}}{l}+3(2+\alpha+\beta)\frac{b_{AB}^2}{l^2}\right]}{1+2\alpha+2\beta+3\alpha\beta}$ | $2EK\left(2+\frac{6\,b_{AB}}{l}+\frac{6\,b_{AB}^2}{l^2}\right)$ | $\frac{2EK(2+3\beta)}{1+2\alpha+2\beta+3\alpha\beta}$ | $4EK$ |
| $\bar{S}_{VAB}$ sidesway stiffness | $\frac{6EK}{l^2}\left(\frac{2+\alpha+\beta}{1+2\alpha+2\beta+3\alpha\beta}\right)$ | $\frac{12EK}{l^2}$ | $\frac{6EK}{l^2}\left(\frac{2+\alpha+\beta}{1+2\alpha+2\beta+3\alpha\beta}\right)$ | $\frac{12EK}{l^2}$ |
| $\bar{M}_{VAB}$ sidesway end moment at $A$ | $-\frac{6EK}{l}\left[\frac{1+\beta+\frac{b_{AB}}{l}(2+\alpha+\beta)}{1+2\alpha+2\beta+3\alpha\beta}\right]$ | $-\frac{6EK}{l}\left(1+\frac{2\,b_{AB}}{l}\right)$ | $-\frac{6EK}{l}\left(\frac{1+\beta}{1+2\alpha+2\beta+3\alpha\beta}\right)$ | $-\frac{6EK}{l}$ |

The slope-deflection equations are simplified when symmetrical conditions of loading and structure exist with respect to any particular member. In such a case $\alpha = \beta$. Furthermore, $b_{AB} = b_{BA} = b$; $M_{RAB} = M_{RBA} = M_R$; $\bar{V}_{AB}' = \bar{V}_{BA}' = V_{AB}' = V_{BA}' = V'$; and the slope-deflection equations corresponding to case I of Table 1 may be reduced to:

$$\bar{M}_{AB} = \frac{2\,E\,K}{1 + 3\,\alpha}\left[\left(\frac{2 + 3\,\alpha}{1 + \alpha} + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_A + \left(\frac{1}{1 + \alpha} + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_B - \left(3 + \frac{6\,b}{l}\right)R\right] - \left(\frac{M_R}{1 + \alpha} + V'\,b\right) \qquad (9a)$$

and

$$\bar{M}_{BA} = \frac{2\,E\,K}{1 + 3\,\alpha}\left[\left(\frac{2 + 3\,\alpha}{1 + \alpha} + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_B + \left(\frac{1}{1 + \alpha} + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_A - \left(3 + \frac{6\,b}{l}\right)R\right] + \left(\frac{M_R}{1 + \alpha} + V'\,b\right) \qquad (9b)$$

For case II of Table 1, considering the semi-rigidity of the joints but neglecting the width of the members, the equations for symmetrical conditions may be reduced to a simple form by letting $b=0$ in Eqs. 9; thus:

$$\bar{M}_{AB} = M_{AB} = \frac{2\,E\,K}{1 + 3\,\alpha}\left[\left(\frac{2 + 3\,\alpha}{1 + \alpha}\right)\theta_A + \left(\frac{1}{1 + \alpha}\right)\theta_B - 3\,R\right] - \frac{M_R}{1 + \alpha} \qquad (10a)$$

and

$$\bar{M}_{BA} = M_{BA} = \frac{2\,E\,K}{1 + 3\,\alpha}\left[\left(\frac{2 + 3\,\alpha}{1 + \alpha}\right)\theta_B + \left(\frac{1}{1 + \alpha}\right)\theta_A - 3\,R\right] + \frac{M_R}{1 + \alpha} \qquad (10b)$$

A similar simplification may be made for case III of Table 1 by letting $\alpha = 0$ in Eqs. 9, in which case the following equations result:

$$\bar{M}_{AB} = 2\,E\,K\left[\left(2 + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_A + \left(1 + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_B - \left(3 + \frac{6\,b}{l}\right)R\right] - (M_R + V_b') \qquad (11a)$$

and

$$\bar{M}_{BA} = 2\,E\,K\left[\left(2 + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_B + \left(1 + \frac{6\,b}{l} + \frac{6\,b^2}{l^2}\right)\theta_A - \left(3 + \frac{6\,b}{l}\right)R\right] + (M_R + V_b') \qquad (11b)$$

*Moment Distribution Applied to Frames with Semi-Rigid Connections.*—The moment-distribution method serves identically the same purpose as the slope-deflection method; that is, the moments at the ends of the individual members

of a frame or continuous beam are determined. It will be assumed, as in the case of the slope-deflection method, that the reader is familiar with the usual moment distribution procedure. The procedure as applied to frames with semi-rigid connections is identical with the usual method, although there are differences in the numerical value of the carry-over, stiffness, and other factors.

The factors used in the moment-distribution procedure may be derived from the slope-deflection equations or by use of the column analogy (11), as will be described herein. Fig. 6 shows the deformation conditions for determining

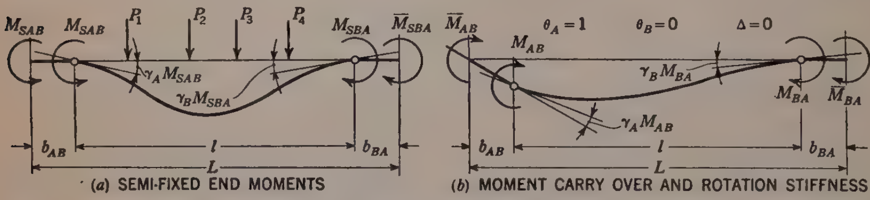



FIG. 6.—DEFORMATION CONDITIONS USED IN MOMENT DISTRIBUTION FOR MEMBERS WITH SEMI-RIGID CONNECTIONS AND FINITE JOINT WIDTH

semi-rigid end moments and the carry-over factor for moment distribution. Table 1 gives the "semi-rigid" end moments, "carry-over" factor, rotation-stiffness factors, sidesway-stiffness factors, and sidesway end moments for the nonsymmetrical cases I, II, III, and IV. Table 2 presents the same factors for the symmetrical cases I, II, and III.

TABLE 2.—MOMENT DISTRIBUTION FOR SYMMETRICAL CASES
$\alpha = \beta$, $b_{AB} = b_{BA} = b$, AND $M_{RA} = M_{RB} = M_R$

| Constant | Symmetrical case I, semi-rigid connections, finite width of members | Symmetrical case II, rigid connections, finite member width | Symmetrical case III, semi-rigid connections, zero member width |
|---|---|---|---|
| $\bar{M}_{SAB}$ semi-fixed-end moment at joint center | $\frac{M_R}{1+\alpha}+V'b$ | $M_R+V'b$ | $\frac{M_R}{1+\alpha}$ |
| $\bar{r}_{AB}$ carry-over factor between joint centers | $\frac{1+6(1+\alpha)\frac{b}{l}+6(1+\alpha)\frac{b^2}{l^2}}{2+3\alpha+6(1+\alpha)\frac{b}{l}+6(1+\alpha)\frac{b^2}{l^2}}$ | $\frac{1+\frac{6b}{l}+\frac{6b^2}{l^2}}{2+\frac{6b}{l}+\frac{6b^2}{l^2}}$ | $\frac{1}{2+3\alpha}$ |
| $\bar{S}_{MAB}$ end rotation stiffness | $\frac{2EK}{1+3\alpha}\left(\frac{2+3\alpha}{1+\alpha}+\frac{6b}{l}+\frac{6b^2}{l^2}\right)$ | $2EK\left(2+\frac{6b}{l}+\frac{6b^2}{l^2}\right)$ | $2EK\left(\frac{2+3\alpha}{1+4\alpha+3\alpha^2}\right)$ |
| $\bar{S}_{VAB}$ sidesway stiffness | $\frac{12EK}{l^2(1+3\alpha)}$ | $\frac{12EK}{l^2}$ | $\frac{12EK}{l^2(1+3\alpha)}$ |
| $\bar{M}_{VAB}$ sidesway end moment at $A$ | $-\frac{6EK}{l}\left(\frac{1+\frac{2b}{l}}{1+3\alpha}\right)$ | $-\frac{6EK}{l}\left(1+\frac{2b}{l}\right)$ | $-\frac{6EK}{l(1+3\alpha)}$ |

All of the moment-distribution factors relate to hypothetical moments at the centers of the joints, and the general procedure is therefore identical with that used in the simple case. After the hypothetical moments at the joint centers are obtained, the moments and shears at the connections result as before from Eqs. 7 and 8.

In applying the moment-distribution method to frame problems, sidesway may be induced either by lateral loads or by unsymmetrical conditions of loading or frame arrangement. The simplest form of sidesway problem is one involving only lateral loads applied to a one-story frame. The lateral load is distributed to the columns in proportion to their "sidesway stiffness" $\bar{S}_V$, and semi-fixed or fixed-end moments are distributed to the ends of each column in proportion to the $\bar{M}_V$ moments for unit sidesway. The next step is the usual moment distribution, but at the conclusion the summation of column shears will not account for the total lateral force. All of the end moments are then multiplied by a constant ratio sufficient to bring the shears into equilibrium with the external lateral force. If the structure is more than one story in height, the procedure is progressively more complicated. Shears are distributed in any one story in proportion to their lateral or sidesway stiffness; but in special cases in which two-story sections adjoin open auditoriums or halls, the combined rigidities of the two stories of columns "in series" must be determined. The stiffness of a two-story group of columns "in series" is given by:

$$\bar{S}_{VABC} = \frac{\bar{S}_{VAB}\,\bar{S}_{VBC}}{\bar{S}_{VAB} + \bar{S}_{VBC}} \qquad \text{(12a)}$$

in which $\bar{S}_{VABC}$ = sidesway stiffness of two columns or groups of columns "in series." The combined rigidity of three tiers of columns "in series" is given by:

$$\bar{S}_{VABCD} = \frac{\bar{S}_{VAB}\,\bar{S}_{VBC}\,\bar{S}_{VCD}}{\bar{S}_{VAB}\,\bar{S}_{VBC} + \bar{S}_{VBC}\,\bar{S}_{VCD} + \bar{S}_{VCD}\,\bar{S}_{VAB}} \qquad \text{(12b)}$$

The analysis of two-story and three-story problems of the foregoing type is taken up in texts (12) (16) (17) and the procedure for frames with semi-rigid joints follows exactly the same course. In applying these analytical methods to the development of design methods for multi-storied buildings, under the action of vertical loads alone, it is reasonable to neglect the effect of sidesway.

Certain short cuts for special conditions may be used, provided that their use in simpler forms of moment distribution is already familiar. For example, if the end $B$ of member $A\,B$ is pin-connected, or freely supported, $\beta$ becomes equal to $\infty$. The "semi-fixed" end moment at $A$ then becomes:

$$\bar{M}_{SAB} = \left(\frac{2 + \frac{2\,b_{AB}}{l}}{2 + 3\,\alpha}\right) M_{RAB} \qquad \text{(13)}$$

In Eq. 13 $M_{RAB}$ is the fixed-end moment at $A$ due to lateral loads in a beam freely supported at $B$. The rotation stiffness, or distribution factor, for end $A$ of member $A\,B$ with $B$ freely supported is:

$$\bar{S}_{MAB} = \frac{2\,E\,K}{2 + 3\,\alpha}\left(3 + \frac{6\,b'_{AB}}{l} + \frac{3\,b_{AB}{}^2}{l^2}\right) \qquad \text{(14a)}$$

The sidesway stiffness factor for the same case, with one end freely supported,

is as follows:

$$\bar{S}_{VAB} = \frac{6\,E\,K}{l^2\,(2 + 3\,\alpha)} \qquad (14b)$$

Cases II and III in Table 2 may be obtained from Eqs. 13, 14*a*, and 14*b* by letting $\alpha$ and $b_{AB}$, respectively, be equal to zero.

Another type of special case occurs when the entire frame and loading upon it are symmetrical. If the center line of the frame is on line with a column, there will be no rotation of the column joints and the center line of the center column may be assumed equivalent to a fixed wall. If there are an odd number of panels, the center line of the frame will cut through the center of the beams in the center bay. The rotation of the two ends of each beam in the center panel will be equal in magnitude and opposite in sign. From this condition it follows that the modified moment stiffness or "distribution factor" may be taken as:

$$\bar{S}_{MAB} = \frac{2\,E\,K}{1 + \alpha} \qquad (14c)$$

for the ends of beams in the center panel. No carry-over is used in the center panel when the modified stiffness factor is used.

*Application of the Column Analogy to Members with Semi-Rigid Connections.*—It will be assumed that the reader is familiar with the application of the column analogy, originally developed by Professor Cross (11), to the determination of moment-distribution factors for beams with variable moments of inertia. The width of the "analogous column" is equal to $\frac{1}{E\,I}$, and the area of any elemental length $ds$ of the analogous column is therefore equal to $\frac{ds}{E\,I}$. From the fundamental relations of the bent beam,

$$\frac{d\phi}{M} = \frac{ds}{E\,I} \qquad (15)$$

in which $d\phi$ = the angle change in any elemental length of beam. At the

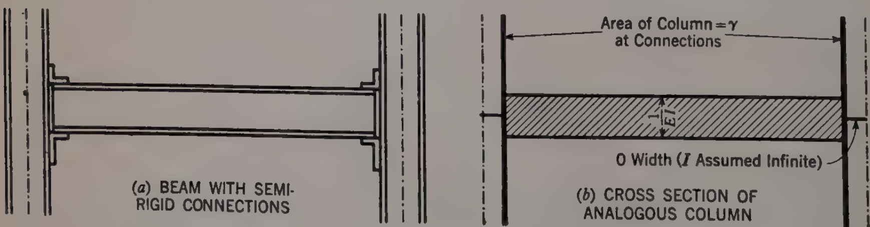



FIG. 7.—APPLICATION OF COLUMN ANALOGY TO BEAMS WITH SEMI-RIGID CONNECTIONS

particular location of the semi-rigid joint, from the definition of the "connection constant,"

$$\gamma = \frac{d\phi}{M} = \frac{ds}{E\,I} \qquad (16)$$

Hence, the localized area of the analogous column at the semi-rigid connection

is equal to the connection constant $\gamma$. Professor Cross (11) has shown that the area of the analogous column at a pin connection is infinite, and in the region of a completely rigid zone it is equal to zero. The semi-rigid connection obviously is a case somewhere between these two extremes, and the column analogy may readily be used to obtain the moment-distribution factors for a member so connected. Fig. 7 illustrates a cross section through the analogous column of a member with semi-rigid end connections.

## Analysis by Slope-Deflection Method

An illustrative example will be presented in detail to demonstrate the application of both the slope-deflection and moment-distribution methods to the analysis of a frame with semi-rigid joints, taking into account the width of the members.

The frame shown in Fig. 8 corresponds to one of the frames actually tested (see Fig. 9), and the connection constant used in the analysis was obtained experimentally from tests of a sample joint. All of the connections were identically alike, and each beam, therefore, was individually symmetrical. The results of the connection test gave an experimental value of $\alpha = 0.01775 \times 10^{-3}$ in inch-kip units. The stiffness of the frame members was measured by bending tests preliminary to fabrication of the frame, and the quantity $E\,I$ was thus found to be $3{,}550 \times 10^3$ and $3{,}321 \times 10^3$ for the beams and columns, respectively, in inch-kip units. The net length $l$ of the beams between connections was 168 in. − 8 in. = 160 in. The columns are continuous, and the beam connections were of the welded seat and top angle type. An approximate correction for column length may be shown to be one third of the beam depth at each end that frames with a beam (see heading "Effect of Width of Member Upon Analysis"). Hence, for the second-story columns, $l = 120 - 6.67 = 113.33$ and, for the first-story columns, $l = 120 - 3.33 = 116.67$. This correction could well be omitted with but little error.

The constant $\alpha$ for the beams was

$$\alpha = \beta = \frac{2\,E\,I\,\gamma}{l} = \frac{2 \times 3{,}550 \times 10^3 \times 0.01775 \times 10^{-3}}{160} = 0.7877.$$

The fixed-end moment for the loading shown is 221.0 in-kips.

Because of the individual symmetry of the beams, the slope-deflection equations in the form of Eqs. 9 were applicable. The typical equation for any beam is written by substituting the values of $\alpha$, $E\,K$, $b$, $l$, etc., in Eqs. 9, which for any loaded beam results in the following:

$$\bar{M}_{AB} = \frac{2 \times 3{,}550}{160(1 + 3 \times 0.7877)}\left\{\left[\left(\frac{2 + 3 \times 0.7877}{1 + 0.7877}\right) + \frac{6 \times 4}{160} + \frac{6 \times 4^2}{160^2}\right]\theta_A\right.$$

$$\left. + \cdots + \left[\left(\frac{1}{1 + 0.7877}\right) + \frac{6 \times 4}{160} + \frac{6 \times 4^2}{160^2}\right]\theta_B\right\} - \left(\frac{221.0}{1.7877} + 6.5 \times 4\right) \quad (17)$$

The right-hand side of this and the following equations has been divided by 1,000 to give more convenient values of $\theta$. The moment $\bar{M}_{AB} = 34.233\,\theta_A$

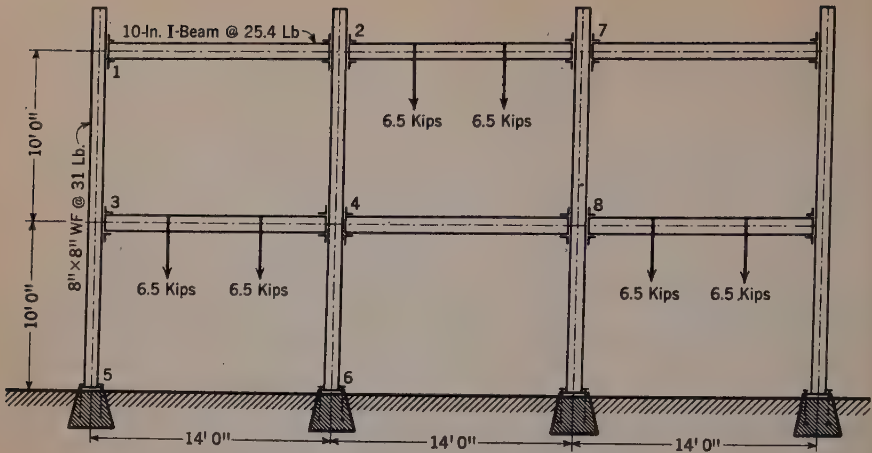



FIG. 8.—TEST FRAME AND ILLUSTRATIVE ANALYSIS

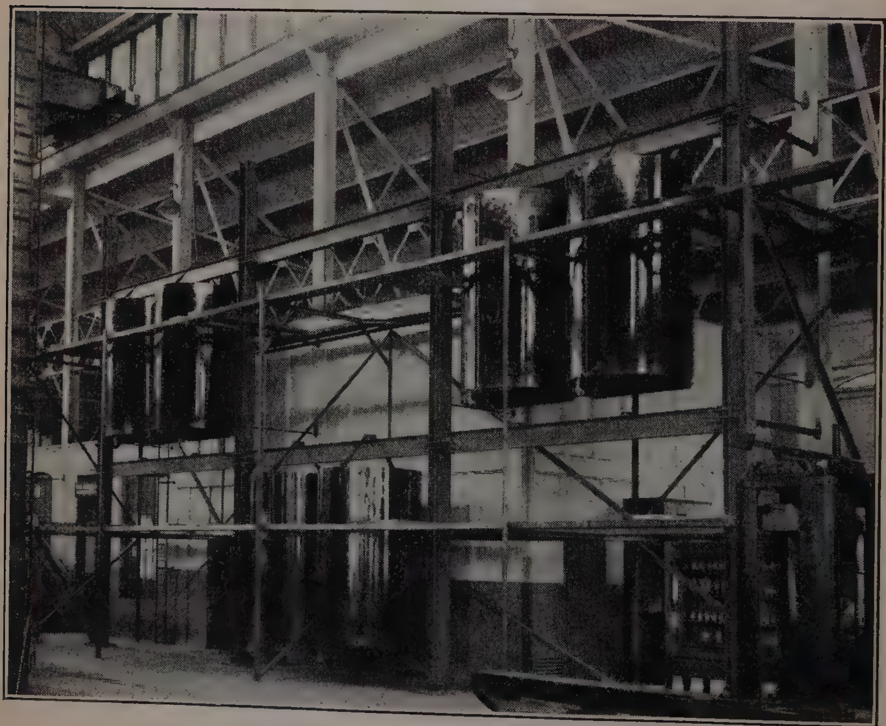


FIG. 9.—VIEW OF TEST FRAME

$+ 9.411\ \theta_B - 149.62$; and similarly:

$$\bar{M}_{BA} = 34.233\ \theta_B + 9.411\ \theta_A + 149.62 \qquad (18)$$

Equations of this type are written for all of the beams in a symmetrical half of the frame, as follows:

$$\left.\begin{aligned}
\bar{M}_{12} &= 34.233\ \theta_1 + 9.411\ \theta_2 \\
\bar{M}_{21} &= 34.233\ \theta_2 + 9.411\ \theta_1 \\
\bar{M}_{27} &= 34.233\ \theta_2 + 9.411\ \theta_7 - 149.62 \quad (-\theta_2 = +\theta_1) \\
&= 24.822\ \theta_2 - 149.62 \\
\bar{M}_{34} &= 34.233\ \theta_3 + 9.411\ \theta_4 - 149.62 \\
\bar{M}_{43} &= 34.233\ \theta_4 + 9.411\ \theta_3 + 149.62 \\
\text{and} \quad \bar{M}_{48} &= 24.822\ \theta_4 \quad (-\theta_4 = +\theta_8)
\end{aligned}\right\} \qquad (19)$$

Similar equations for the column moments are written by making the proper substitutions in slope-deflection Eqs. 11, as follows:

$$\left.\begin{aligned}
\bar{M}_{13} &= 127.865\ \theta_1 + 69.257\ \theta_3 \\
\bar{M}_{24} &= 127.865\ \theta_2 + 69.257\ \theta_4 \\
\bar{M}_{31} &= 127.865\ \theta_3 + 69.257\ \theta_1 \\
\bar{M}_{42} &= 127.865\ \theta_4 + 69.257\ \theta_2 \\
\bar{M}_{35} &= 123.910\ \theta_3 \quad (\theta_5 = 0) \\
\bar{M}_{46} &= 123.910\ \theta_4 \quad (\theta_6 = 0) \\
\bar{M}_{53} &= 61.810\ \theta_3 \quad (\theta_5 = 0) \\
\text{and} \quad \bar{M}_{64} &= 61.810\ \theta_4 \quad (\theta_6 = 0)
\end{aligned}\right\} \qquad (20)$$

The sidesway is obviously zero because of symmetry, and the only unknowns are the four angle changes $\theta_1$, $\theta_2$, $\theta_3$, and $\theta_4$. The necessary and sufficient conditions for the solution are obtained by applying the joint equilibrium equation $\Sigma M = 0$ to the four joints 1, 2, 3, and 4:

$$\left.\begin{aligned}
\bar{M}_{12} + \bar{M}_{13} &= 0 \\
\bar{M}_{21} + \bar{M}_{24} + \bar{M}_{27} &= 0 \\
\bar{M}_{31} + \bar{M}_{34} + \bar{M}_{35} &= 0 \\
\text{and} \quad \bar{M}_{43} + \bar{M}_{42} + \bar{M}_{48} + \bar{M}_{46} &= 0
\end{aligned}\right\} \qquad (21)$$

Rewriting these equations in terms of the unknown $\theta$'s:

$$\left.\begin{array}{rrrrl}
+162.052\ \theta_1 + & 9.411\ \theta_2 + & 69.214\ \theta_3 & & = 0 \\
9.411\ \theta_1 + & 186.874\ \theta_2 + & & + 69.214\ \theta_4 & = +149.620 \\
+69.214\ \theta_1 & & +285.934\ \theta_3 + & 9.411\ \theta_4 & = +149.620 \\
 & +69.214\ \theta_2 + & 9.411\ \theta_3 + & 310.756\ \theta_4 & = -149.620
\end{array}\right\} \qquad (22)$$

The solution of these four simultaneous equations may be made by systematic elimination of unknowns (15) (16) or by a method of successive approximations (15). The following solution was obtained by the first method: $\theta_1 = -0.33165$;

$\theta_2 = +1.09286$; $\theta_3 = +0.62791$; and $\theta_4 = -0.74389$. These values of $\theta$ are 1,000 times the actual values but will give the correct moments when substituted in the moment equations which previously had been divided by 1,000. The actual moments at the connections may be found from the hypothetical joint center moments by computing the shears with Eqs. 7 and the connection moments with Eqs. 8. An alternate method would be to construct, graphically, the simple beam moment diagram for the full lengths $L$ upon the joint moment base line. The connection moments then could be scaled off as the ordinate to the moment diagram at the face of the connecting member. Table 3 gives the results by the analytical method.

TABLE 3.—COMPUTATION BY SLOPE DEFLECTION

| Location, joint and member | Joint moment $\bar{M}$ from slope-deflection equation | $\bar{V}$ by Eqs. 7 | Connection moment $M$ by Eqs. 8 |
|---|---|---|---|
| 1–2 | − 1.07 | −0.198 | − 1.86 |
| 2–1 | + 34.29 | −0.198 | + 33.50 |
| 2–7 | −122.49 | +6.500 | − 96.49 |
| 3–4 | −135.12 | +6.531 | −109.00 |
| 4–3 | +130.06 | −6.469 | +104.18 |
| 4–8 | − 18.46 | +6.500 | + 7.54 |
| 1–3 | + 1.07 | −0.515 | − 0.65 |
| 3–1 | + 57.31 | −0.515 | + 55.59 |
| 3–5 | + 77.79 | −1.000 | + 74.46 |
| 5–3 | + 38.82 | −1.000 | + 38.82 |
| 2–4 | + 88.20 | −0.607 | + 86.18 |
| 4–2 | − 19.44 | −0.607 | − 21.46 |
| 4–6 | − 92.15 | +1.186 | − 88.20 |
| 6–4 | − 45.99 | +1.186 | − 45.99 |

## Analysis by Moment Distribution

The factors required in the moment-distribution procedure have been presented in Tables 1 and 2. Some of the necessary computations in the following have already been made under the heading "Analysis by Slope-Deflection Method":

*Semi-Fixed End Moment at Joint Center.*—

$$\bar{M}_{SAB} = \frac{221.0}{1.7877} + 6.5 \times 4 = 149.62 \text{ in-kips.}$$

*Carry-Over Factors Between Joint Centers.*—

Beams (Both Directions).—

$$\bar{r}_{ab} = \bar{r}_{ba} = \frac{1 + 6\,(1.7877)\,\frac{4}{160} + 6\,(1.7877)\,\frac{4^2}{160^2}}{2 + 3\,(0.7877) + 6\,(1.7877)\,\frac{4}{160} + 6\,(1.7877)\,\frac{4^2}{160^2}} = 0.275.$$

Second-Story Columns (Both Directions).—

$$r = \frac{1 + 6\left(\frac{3.33}{113.33}\right) + 6\left(\frac{3.33^2}{113.33^2}\right)}{2 + 6\left(\frac{3.33}{113.33}\right) + 6\left(\frac{3.33^2}{113.33^2}\right)} = 0.542.$$

First-Story Columns (Top to Bottom) (See Table 1).—

$$r = \frac{1 + 3\left(\frac{3.33}{116.67}\right)}{2 + 6\left(\frac{3.33}{116.67}\right) + 6\left(\frac{3.33}{116.67}\right)^2} = 0.499.$$

*End Rotation Stiffness at Joint Centers.*—

Beams (End Bay).—

$$\bar{S}_M = \frac{2\ (3{,}550) \times 10^6}{160\ [1 + 3\ (0.7877)]} \left[\frac{2 + 3\ (0.7877)}{1.7877} + 6\left(\frac{4}{160}\right) + 6\left(\frac{4^2}{160^2}\right)\right] = 34.2 \times 10^6.$$

Beams (Modified Stiffness in Center Bay Due to Symmetry Requiring Analysis of Only One Half of Frame—Eq. 14*c*).—

$$\bar{S}_M = \frac{2\ (3{,}550) \times 10^6}{160\ (1 + 0.7877)} = 24.8 \times 10^6.$$

Second-Story Columns.—

$$\bar{S}_M = \frac{2\ (3{,}321) \times 10^6}{113.33}\left[2 + 6\left(\frac{3.33}{113.33}\right) + 6\left(\frac{3.33^2}{113.33^2}\right)\right] = 127.9 \times 10^6.$$

First-Story Columns (Upper End).—

$$\bar{S}_M = \frac{2\ (3{,}321) \times 10^6}{116.67}\left[2 + 6\left(\frac{3.33}{116.67}\right) + 6\left(\frac{3.33}{116.67}\right)^2\right] = 123.9 \times 10^6.$$

Proportional factors for distributing moments to ends of members are given in Table 4. The distribution may be done either directly on a diagram

TABLE 4.—Proportional Factors for Distributing Moments to Ends of Members

| Joint 1 | | | Joint 2 | | | Joint 3 | | | Joint 4 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Member | End rotation stiffness | Distribution factor | Member | End rotation stiffness | Distribution factor | Member | End rotation stiffness | Distribution factor | Member | End rotation stiffness | Distribution factor |
| 1–3 | 127.9 | 0.789 | 2–1 | 34.2 | 0.182 | 3–5 | 123.9 | 0.433 | 4–3 | 34.2 | 0.110 |
| 1–2 | 34.2 | 0.211 | 2–4 | 127.9 | 0.685 | 3–1 | 127.9 | 0.447 | 4–2 | 127.9 | 0.411 |
| ... | ..... | ..... | 2–7 | 24.8 | 0.133 | 3–4 | 34.2 | 0.120 | 4–8 | 24.8 | 0.080 |
| ... | ..... | ..... | ... | ..... | ..... | ... | ..... | ..... | 4–6 | 123.9 | 0.399 |
| ... | 162.1 | 1.000 | ... | 186.9 | 1.000 | ... | 286.0 | 1.000 | ... | 310.8 | 1.000 |

of the frame in the manner frequently followed or in tabular form. The solution is herein presented in tabular form (see Fig. 10), through five cycles after the initial distribution. Each cycle consists successively of: (*a*) The carry-over of moments from the previously distributed moments; and (*b*) the distribution of the new unbalanced moment at each joint to the ends of the members. The final summation of moments may be compared with the results of the solution by the slope-deflection method, and the results are seen to check with a maximum error of two in the third significant figure, or a fraction of 1%, except in the case of the smallest moment of 1.07 in-kips,

when the error is about 2%. The moments resulting from the distribution procedure are hypothetical moments at the joint center, and the actual moments at the connection may be found by the method previously described.

Carry-Over Factor 0.275

| 1·3 | 1·2 |
|---|---|
| 0.789 | 0.211 |
| 0 | 0 |
| 0 | 0 |
| +36.52 | + 7.49 |
| −34.51 | − 9.23 |
| + 1.10 | + 1.67 |
| − 2.19 | − 0.58 |
| + 4.98 | + 0.80 |
| − 4.56 | − 1.22 |
| + 0.38 | + 0.15 |
| − 0.42 | − 0.11 |
| + 0.64 | + 0.09 |
| − 0.58 | − 0.15 |
| + 1.09 | − 1.09 |

| 2·1 | 2·4 | 2·7 | ←Location |
|---|---|---|---|
| 0.182 | 0.685 | 0.133 | ←Distribution Factors |
| 0 | 0 | −149.62 | ←Semi-Rigid End Moments |
| + 27.23 | +102.49 | + 19.90 | Distribution |
| 0 | − 33.33 | 0 | Carry Over |
| + 6.70 | + 22.83 | + 4.43 | Distribution |
| − 2.54 | − 13.47 | 0 | Carry Over |
| + 2.91 | + 10.97 | + 2.13 | Distribution |
| − 0.16 | − 2.79 | 0 | Carry Over |
| + 0.54 | + 2.02 | + 0.39 | Distribution |
| − 0.34 | − 1.47 | 0 | Carry Over |
| + 0.33 | + 1.24 | + 0.24 | Distribution |
| − 0.03 | − 0.25 | 0 | Carry Over |
| + 0.05 | + 0.19 | + 0.04 | Distribution |
| + 34.06 | + 88.43 | −122.49 | Summation of Moments |

0.542 ← Carry-Over Factor 0.275 0.542 ← Carry Over (Columns)

| 3·5 | 3·1 | 3·4 |
|---|---|---|
| :0.433 | 0.447 | 0.120 |
| 0 | 0 | −149.62 |
| +64.79 | +66.88 | + 17.95 |
| 0 | 0 | − 4.53 |
| + 1.96 | + 2.03 | + 0.54 |
| 0 | −18.70 | − 1.83 |
| + 8.89 | + 9.18 | + 2.46 |
| 0 | − 1.19 | − 0.38 |
| + 0.68 | + 0.70 | + 0.19 |
| 0 | − 2.47 | − 0.20 |
| + 1.16 | + 1.19 | + 0.32 |
| 0 | − 0.23 | − 0.04 |
| + 0.12 | + 0.12 | + 0.03 |
| +77.60 | +57.51 | −135.11 |

| 4·3 | 4·2 | 4·8 | 4·6 | ←Location |
|---|---|---|---|---|
| 0.110 | 0.411 | 0.080 | 0.399 | ←Distribution Factors |
| +149.62 | 0 | 0 | 0 | ←Semi-Rigid End Moments |
| − 16.46 | − 61.49 | − 11.97 | −59.70 | Distribution |
| + 4.94 | + 55.55 | 0 | 0 | Carry Over |
| − 6.65 | − 24.86 | − 4.84 | −24.14 | Distribution |
| + 0.15 | + 12.37 | 0 | 0 | Carry Over |
| − 1.38 | − 5.15 | − 1.00 | − 4.99 | Distribution |
| + 0.68 | + 5.95 | 0 | 0 | Carry Over |
| − 0.73 | − 2.72 | − 0.53 | − 2.65 | Distribution |
| + 0.05 | + 1.09 | 0 | 0 | Carry Over |
| − 0.13 | − 0.47 | − 0.09 | − 0.45 | Distribution |
| + 0.09 | + 0.67 | 0 | 0 | Carry Over |
| − 0.08 | − 0.32 | − 0.06 | − 0.30 | Distribution |
| +130.10 | − 19.38 | − 18.49 | −92.23 | Summation |

0.499 0.499 ← Carry Over to Fixed Bases. (One Process)

5·3 $\boxed{+38.72}$ 6·4 $\boxed{-46.02}$ Moments at Column Bases

FIG. 10.—SOLUTION BY MOMENT DISTRIBUTION

## SIDESWAY INDUCED BY UNSYMMETRICAL VERTICAL LOADS

The writers have analyzed the frame shown in Fig. 11 by both slope deflection and moment distribution in order to study the effect of neglecting sidesway as induced by unsymmetrical vertical loads. Space does not permit the details of the analysis, which follows usual procedures, however. The dimensions of the frame and size of members are shown in Fig. 11 and the beam *a b* is assumed to carry a uniformly distributed load of one kip per foot. As in the previous case, it was assumed that the columns were fixed at the base, but the beam-column connections were assumed to have "50% rigidity," which corresponds to $\alpha \doteq 1$. The results are presented in Table 5.

Although no general conclusions should be drawn from this single case, it is seen that in Table 5 sidesway could have been neglected without great

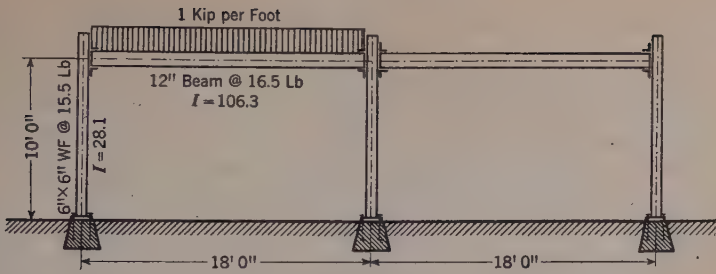



FIG. 11.—UNSYMMETRICALLY LOADED FRAME

error in the end moments. Sidesway due to vertical loads usually will be less in frames with semi-rigid connections as compared with the same frames rigidly connected, and will be further decreased in the actual structure by walls and concrete incasement.

TABLE 5.—SIDESWAY INDUCED BY UNSYMMETRICAL VERTICAL LOADS

| Joint and member | By Moment Distribution | | | By slope deflection: Moment at joint center by slope-deflection method | Connection Moments | | |
|---|---|---|---|---|---|---|---|
| | Moments at joint centers, neglecting sidesway | Sidesway moment to balance shear | Moment at joint center, corrected for sidesway | | *b*, in in. | Shear at end of member by Eqs. 7 | Moment at face of connecting member |
| *a d* | +119.02 | − 6.74 | +112.28 | +112.32 | 4 | −1.34 | +106.96 |
| *d a* | + 59.32 | −10.53 | + 48.79 | + 48.80 | 0 | −1.34 | + 48.80 |
| *a b* | −119.02 | + 6.74 | −112.28 | −112.32 | 3 | +8.59 | − 86.52 |
| *b a* | +141.71 | + 5.10 | +146.81 | +146.85 | 3 | −8.91 | +120.12 |
| *b e* | − 88.61 | −10.20 | − 98.81 | − 98.88 | 4 | +1.29 | − 93.72 |
| *e b* | − 44.16 | −12.25 | − 56.41 | − 56.42 | 0 | +1.29 | − 51.26 |
| *b c* | − 53.10 | + 5.10 | − 48.00 | − 47.97 | 3 | +0.23 | − 47.28 |
| *c b* | − 7.63 | + 6.74 | − 0.89 | − 0.89 | 3 | +0.23 | − 0.20 |
| *c f* | + 7.63 | − 6.74 | + 0.89 | + 0.89 | 4 | +0.05 | + 1.09 |
| *f c* | + 3.80 | −10.53 | − 6.73 | − 6.72 | 0 | +0.05 | − 6.52 |

## EFFECT OF WIDTH OF MEMBER UPON ANALYSIS

In the analysis of frames, the length of each member is often assumed to be equal to the distance center-to-center of joints. The moments thus computed at the joint centers will usually be higher than the actual moment at the connection at the end of the member. This method of computation is usually on the safe side in determining end-connection moments but generally will be on the unsafe side in determining the maximum positive moment near the center of the beam.

An approximate correction is sometimes made for the effect of member width. The end moments computed in the foregoing manner are used to construct the moment diagram. The actual end-connection moment to be used in design is then taken as the ordinate to the moment diagram at the face of the column or connecting member. This method usually gives values of end-connection moments that are too low.

The error by either of the foregoing methods becomes greater as the ratio between the width of the joint and the length of the member increases. The

average errors are also greater for frames with semi-rigid connections than for frames with rigid connections.

In order to develop criteria to determine when, and when not, to consider member width in analysis, the behavior of the frame shown in Fig. 12 was studied for various ratios of joint width to member length. The load was assumed to act uniformly on beams 1–2 and 3–4, and the analyses were made for four different ratios of joint width to member length—namely, $\frac{1}{15}$, $\frac{1}{12}$, $\frac{1}{9}$, and $\frac{1}{6}$. Analyses also were made neglecting member width entirely, and the method of arbitrary correction previously outlined was tried also. The

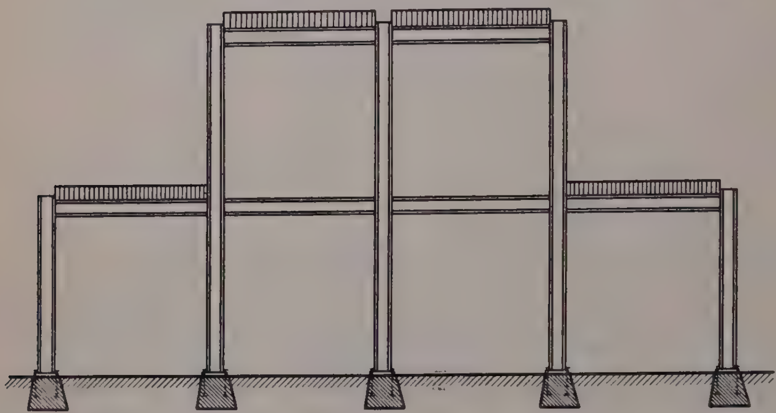


Fig. 12

analyses were made both for a frame with rigid connections and for a frame with continuous columns but with semi-rigid beam-to-column connections. All of the analyses were made by the method of moment distribution.

Special note should be made of the length of the columns in relation to beam depth. The methods herein presented to take account of width of joint are based on the assumption that the interior of the joint may be considered infinitely rigid in comparison with the bending stiffness of the member. In the case of a beam framing into a column, this assumption seems reasonable, particularly if the column runs through the joint without a splice. In the case of the continuous column, however, the connection moments are introduced by concentrated lateral forces acting at the top and bottom of the beam in the type of connection shown in Fig. 2(*b*). In such a case it may be shown that nearly correct results may be obtained for the moments in the column by assuming a length correction for the column of one third the beam depth at each end instead of one half the beam depth. This correction was made in the analyses under consideration and was found to give good results in actual frame tests.

The results of these studies are shown in Fig. 13 for the frames with rigid and semi-rigid connections, respectively. The solid lines give the percentage of error of moments determined with a neglect of joint width as compared with

corresponding moments correctly computed at the face of the joint. The broken lines give the percentage of error resulting from the arbitrary correction for joint width by neglecting it in the analysis but using the ordinate of the moment diagram at the face of the joint.

It is noted in Fig. 13 that the maximum percentage of error occurs in the case of the large end moments in the loaded beams at joints 1 and 4. It also

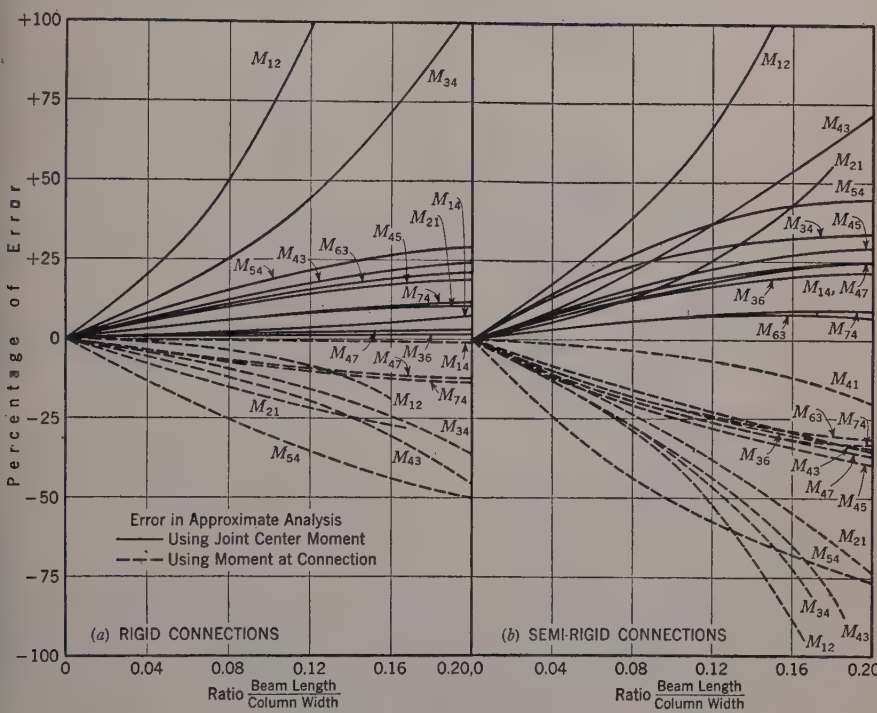



FIG. 13.—PERCENTAGE OF ERROR IN CONNECTION MOMENT, NEGLECTING MEMBER WIDTH

may be seen that the errors are usually larger in the frame with semi-rigid connections than in the frame with rigid connections. The errors are appreciable even for low ratios of joint width to beam length. In the case of a one-to-twenty ratio, for example, the error may be as high as 20%, with the average error about 5% for the rigid frame and as high as 25%, and with an average error close to 10% for the frame with semi-rigid connections. As the ratio of joint width to beam length increases, the errors become increasingly larger.

A fairly close approximation for the moment at the connection is obtained by neglecting joint width in the analysis and using, as the connection moment, the moment halfway between the connection and joint center.

## COMPARISON BETWEEN THEORETICAL ANALYSES AND TEST RESULTS

In order to compare the results of analyses with the actual behavior of building frames, two full-size, all-welded, model building frames were con-

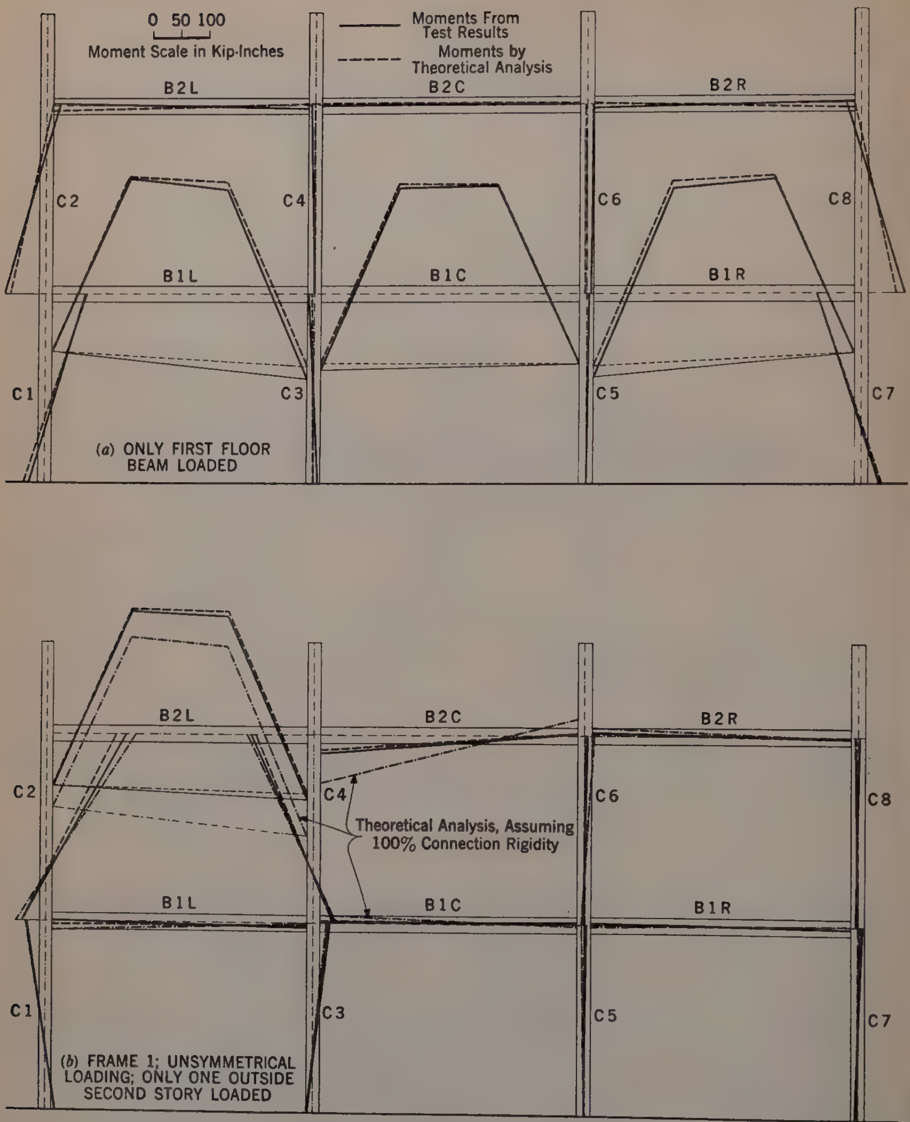
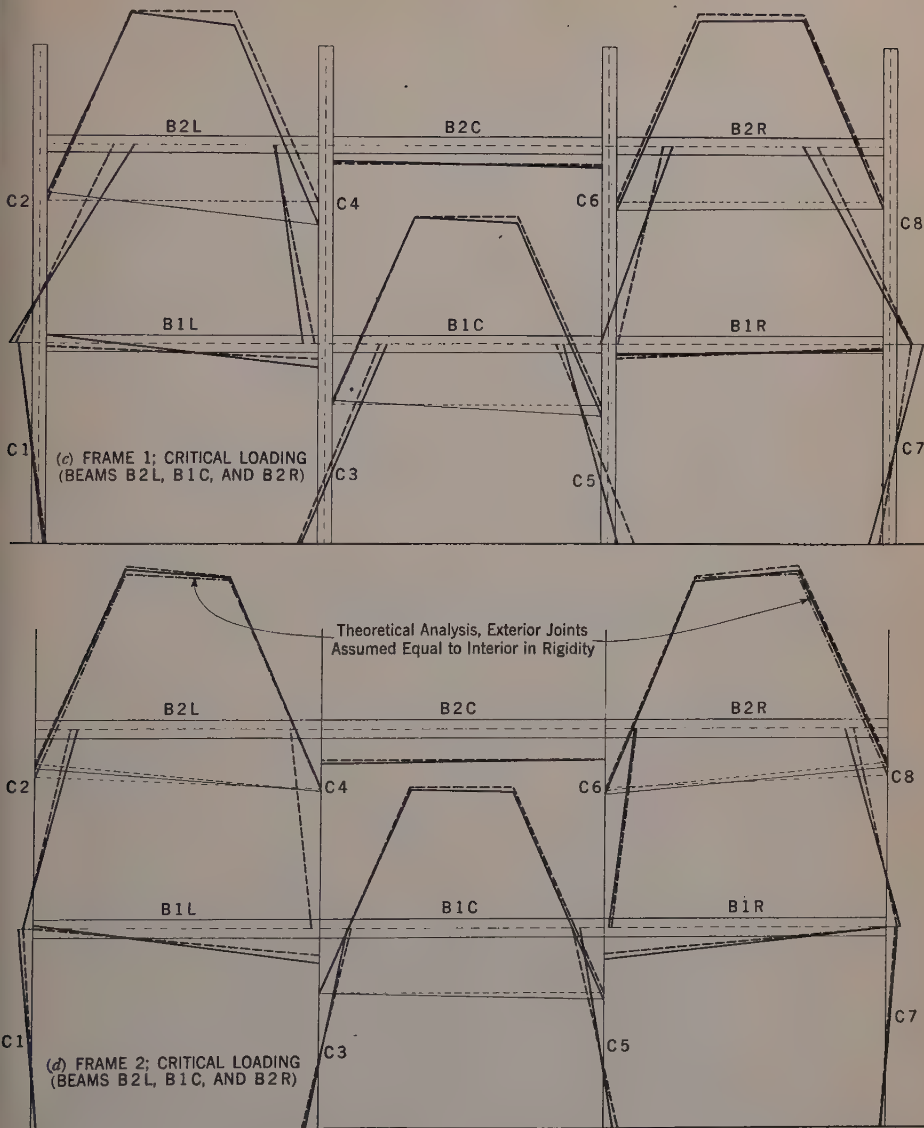



FIG. 14.—COMPARISON OF COMPUTED AND




OBSERVED MOMENTS, FOR CRITICAL CONDITIONS

structed. Details of these tests have already been presented in another paper by the writers (2).

Frame No. 1 was made with beam-to-column flange connections, whereas frame No. 2 had beam-to-column web connections. The general dimensions and size of members of frame No. 1 are shown in Fig. 8 in connection with the illustrative example (see heading "Analysis by Slope-Deflection Method"), and a photograph of the same frame is shown in Fig. 9. The beam-to-column connection used in these frames consisted of welded seat and top angles, the details and semi-rigid properties of which have been described elsewhere (2). Vertical loads were applied to the frames by means of water tanks, which are shown in Fig. 9 in one of the loading positions. Each frame was braced laterally near each joint by means of flexible ties welded between columns of the frame and columns of the laboratory. These ties had reduced sections near each end that allowed the frame full freedom to bend or move laterally in its own plane but that prevented movement out of its own plane.

The computation of the moments developed during tests of the frames was made by measuring the rotation at the ends of each beam and at the joint centers by means of the 20-in. level bar which was illustrated in Fig. 3. Then the moments at the end of each beam and column could be calculated by the slope-deflection equations (see Eqs. 3).

The connection constants for typical joints in the frame were determined by means of the setup shown in Fig. 2($b$). The experimentally determined values of these connection constants, as determined by Fig. 4, have been used in the theoretical analyses. The method of moment distribution was used and a typical analysis, taking account of the width of member, has been presented in the illustrative example.

Fig. 14 shows both the computed and experimentally determined moments for several of the critical conditions of load that were applied to the two frames. Fig. 14($a$) shows moment diagrams for frame No. 1 with only first-floor beam loaded. Fig. 14($b$) is for frame No. 1 with unsymmetrical loading in which only one outside second-story beam was loaded. Sidesway was neglected in the analysis but the agreement between analysis and experimental result is excellent. A comparison is made in this case with an analysis assuming completely rigid joints. The actual test results agree well with the analysis for semi-rigid joints but are widely divergent from the analysis for rigid points. It should be noted that the moments "taper out" much more rapidly in a frame with semi-rigid connections than in one with rigid joints. Fig. 14($c$) is for a critical condition of loading. In applying the test load for this case, the order of loading was purposely unbalanced but the moments by test are in fairly good agreement with the theoretical analysis. Fig. 14($d$) is for frame No. 2, with beam-to-column web connections, and is for the same critical loading condition as Fig. 14($c$). The outside column connections in frame No. 2 has less rigidity than the inside, and this was taken into account in the analysis. The analysis based on the assumption that the outside joints are as rigid as

the interior joints is also given, and it is seen that the test results usually fall between the two different analyses.

In general, the test results agree well with the methods of analysis which have been presented. The results also show that the test of a single joint to determine the connection constant gives a satisfactory measure of the behavior of the same type of joint used in an actual frame.

## The Design of Frames for Partial Rigidity

The methods of analysis which have been presented in this paper obviously are not directly applicable to design. Any method of statically indeterminate analysis requires an assumed structure as a preliminary to design. To assume a building design, and then to analyze such a highly redundant structure by the methods that have been presented, would be an impractical design procedure, warranted only for very special problems.

For routine building design, a suitable method must be direct and simple in application. Such design methods have been presented by the writers in conjunction with a particular type of all-welded, beam-to-column connections (2), and in a more general article covering the application to any semi-rigidly connected structure (3). The British Steel Structures Research Committee (1) has developed design procedures for frames with semi-rigid riveted connections. In a letter dated December 26, 1940, S. D. Lash, secretary of the Subcommittee on Steel Construction, National Building Code, National Research Council of Canada, stated that simplifications in the original design method have been made in Great Britain and that similar steps are in progress in Canada.

The design procedure for the beams in welded building frames with semi-rigid connections that has been developed by the writers (2) may be outlined as follows:

1. The beams are designed by the usual procedure of computing the required section modulus for maximum simple beam moment.
2. The section modulus for maximum simple beam moment is multiplied by a reduction factor that depends on the distribution of load and relative stiffness of the simple beam and adjacent column sections. This reduction factor is obtained from a graph or simple formula and is based on the most critical combination of load possible.
3. The final beam selection is determined by the reduced section modulus found by step 2.

Although the method was developed for designing beams with welded connections, it is applicable to any frame having connections with the desired semi-rigid properties.

In computing the reduction factor, the stiffening effect of adjacent beams was neglected, and the same formula applies to exterior and interior bays. By this procedure, with end connections designed for 50% end restraint, an

average saving in the weight of beams of between 15% and 20% was found possible. If greater refinement and complexity are introduced into the design procedure, the average saving in weight of beams might be raised to more than 20%.

## Conclusion

The methods presented and corroborated by test in this paper represent a refinement in the analysis of building frames. It may be questioned whether such refinement is warranted. The concrete encasement of beams, columns, walls, and partitions, and the uncertainties of applied load, all represent indeterminate quantities which, undoubtedly, may have as great, or greater, effect upon frame behavior as does the semi-rigidity of the bare steel connection. Nevertheless, discounting these uncertainties as assets that cannot be counted upon definitely, there remains the certain dependable bare connection end-restraint. This influence can be determined and applied to the development of improved and more economical methods of design.

## Acknowledgment

The tests of the all-welded steel building frame were part of an investigation conducted at the Fritz Engineering Laboratory of Lehigh University, at Bethlehem, Pa., in cooperation with the Structural Steel Welding Committee of the American Welding Society. Leon S. Moisseiff, M. Am. Soc. C. E., was chairman of the committee. The writers received the cooperation of Prof. Hale Sutherland, M. Am. Soc. C. E., director of the Fritz Engineering Laboratory, and Howard J. Godfrey, engineer of tests.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

# ANALYSIS OF LEGAL CONCEPTS OF SUBFLOW AND PERCOLATING WATERS

## Discussion

BY C. F. TOLMAN AND AMY C. STIPP

C. F. TOLMAN[82] AND AMY C. STIPP[83] (by letter).[83a]—The writers have received both benefit and pleasure from the discussions of their paper. These discussions have been in two forms: Formal written contributions published in *Proceedings*, and informal discussion by personal letter or conversation.

The personal discussions were more critical than those published, and showed widely divergent viewpoints. For example, an engineer criticized the paper because of its limited scope, and opinion was expressed that the value of the contribution was greatly reduced by the lack of a discussion of confined water. On the other hand, a lawyer thought the paper covered far too large a field, and suggested that the writers limit discussion to the "support" of stream flow by ground water.

The writers appreciate that only one small phase of the application of ground-water hydrology to legal concepts was treated, and that the hydrology of confined water and analysis of the decisions regarding it are of increasing importance to the engineer and lawyer.

In both the personal discussions and published discussions, the presentation of the effects of pumping on influent and effluent streams was criticized as incomplete. The writers attempted to show that pumping influent subflow not in contact with the stream would not affect surface stream flow as far as influent conditions extend; and that a contraction of stream channel farther downstream would force to the surface a subflow diminished by pumping and thus indicate in the diminished surface flow that water had been removed from subflow.

The writers did not mention the case of a pump situated at a narrow part of the stream channel, with enlargement of stream gravels above and below the

NOTE.—This paper by C. F. Tolman and Amy C. Stipp was published in December, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1940, by Donald M. Baker, M. Am. Soc. C. E.; April, 1940, by Messrs. Samuel C. Wiel, Hyde Forbes, and Ronald B. Harris; May, 1940, by Messrs. Edward F. Treadwell, O. E. Meinzer, M. R. Lewis, and Bayard F. Snow; and June, 1940, by Harold Conkling, M. Am. Soc. C. E.

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[83] Tucson, Ariz.

[83a] Received by the Secretary January 27, 1941.

narrows. No increased seepage loss due to pumping is registered by measurement of influent seepage below the narrows. Above the narrows, pumping might change effluent conditions, produced by channel contraction, to influent conditions, and measurements of stream seepage loss above the narrows and the pumping plant would register a diminution of surface flow above the narrows.

The writers approve most of the points made in the discussions and, furthermore, many valuable features beyond the scope of the paper were introduced. Only some of the important contributions, and a few of the points on which there is disagreement, will be mentioned.

Mr. Baker mentions that difficulties in California procedure due to faulty legal concepts of ground-water hydrology still exist because many cases were tried and many decisions rendered during the dry cycle at the turn of the century, and that the advances in ground-water hydrology from 1920 to 1940 have not as yet modified the erroneous concepts on which the earlier decisions were founded.

He presents an excellent mathematical method of classifying ground-water movement according to the degrees of confinement of ground-water flow. He attributes many of the recent cases in Southern California to opposition to the lowering of ground-water level by those claiming a superior right to ground water to those possessing inferior rights. He rightfully emphasizes that this demand that the water table must not be lowered may be as absurd as prohibiting the lowering of water level in a surface reservoir during a dry period.

Mr. Wiel's contribution is undoubtedly the most valuable discussion of legal phases of the application of ground-water hydrology. The writers appreciate the exhaustive research that he has made on the subject of ground-water law, and are especially pleased that he has presented so clearly the difficulties in apportioning water according to priority of appropriation, because the doctrine of rights by appropriation has received support by many engineers and lawyers.

Mr. Wiel shows great skill in comparing statements of two authorities on the same subject, attempting to prove, thereby, disagreement in general principles. As a matter of fact, the apparent disagreement usually lies in the difficulty of discussing a complex subject in precise language, with a careful definition of conditions under which the statement holds true. For example, the writers' statement that ground water never occurs as a stationary water body is compared with Mr. Meinzer's suggestion[30] that "some of the water may be virtually stationary in synclinal troughs or encased lenses." This suggestion of Mr. Meinzer is of very limited application, whereas the writers' statement applies to all bodies of ground water in pervious alluvial material.

Mr. Wiel quotes Messrs. Baker and Conkling's statement that the proper conception of ground water in alluvium is that of water in a reservoir, whereas the writers emphasize objections to this concept. Mr. Wiel quotes the senior writer's statement[32] that no water table can exist in impervious material, whereas Mr. Meinzer indicates that in most cases no material is absolutely

[30] "Groundwater in the United States," by O. E. Meinzer, *U. S. Geological Survey Water Supply Paper, No. 836–D*, 1939, p. 182.

[32] "Ground Water," by C. F. Tolman, McGraw-Hill Book Co., Inc., 1937, p. 225.

impervious. "Impervious" and "relatively impervious" have been used more loosely than almost any other hydrologic terms.

Mr. Harris' approval of the scientific foundation, not only for legal decisions but in the preparation of cases, is especially valuable. In the next to the last paragraph he states:

> "The trend in the law following the development of large areas of arid lands through irrigation, with its accompanying creation of new and changed water tables, requires thorough and accurate knowledge of ground water, not only in settling disputes as to the rights to use of such ground water, but in its control for the beneficial and most economical use, and in the preservation of lower lying lands from destruction by too high a water table. For this courts and water users must look to irrigation engineers. The fountainhead for their learning is the universities. Are the universities fulfilling this requirement in the education of their engineers who desire to specialize in irrigation? Are these engineers being qualified as experts in ground water? It is the understanding of the writer that, generally, they are not."

The writers believe that unfortunately engineering instruction in most colleges does not treat, adequately, the laws and principles governing the occurrence and motions of underground water.

Mr. Treadwell criticizes three cases cited for discussion by the writers. An introductory paragraph to this criticism is as follows:

> "Practically every case cited is attacked as being in some way in violation of the positive rules of hydrology developed by the paper. This probably indicates that the scientist has as much difficulty in understanding a judicial decision as the lawyer has in stating a scientific principle. Such a paper would be of much more value if it had been prepared in cooperation with a lawyer. The writer disagrees entirely with the criticisms made of the decisions in question."

The writers assure Mr. Treadwell that they made every attempt to secure the cooperation of an attorney versed in ground-water law, but were not entirely successful.

Mr. Treadwell takes exception to the writers' criticism of the decision in Maricopa County Municipal Water Conservation District No. 1 vs. Southwest Cotton Company,[20] which may have been somewhat biased and harsh. The senior writer testified in the trial of this case, and, in spite of attempts to maintain the independent position of a scientist, his personal experience may have been reflected in his comments on the decision. However, the discussion and principal criticism were directed to the legal "test" or "rule" set forth by the judge to determine the existence of a subsurface stream. Even a vivid geological imagination cannot conceive of so perfect an equilibrium that pumping from subflow abstracts exactly the same quantity of water from the surface flow.

The discussion of two cases, Vineland Irrigation District vs. Azusa Irrigation Company,[21] and Lemm vs. Rutherford,[22] shows that the writers were quite unsuccessful in explaining the behavior between influent and effluent streams and

[20] 39 Ariz. 65, 367, 4 Pac. (2) 369 (1931), 7 Pac. (2) 254 (1932).

[21] Vineland Irrigation District vs. Azusa Irrigation Co., 126 Cal. 486, 58 Pac. 1057, 46 L. R. A. 820 (1899).

[22] Lemm vs. Rutherford, 76 Cal. App. 455, 245 Pac. 225 (1926).

the water tables adjacent thereto. The latter case involved the right to construct a sump in which seepage from a nearby ditch was collected, and from which the water was taken. As seepage was taking place the ditch was influent. As stated in the paper, if the mound developed by seepage water is not in contact with the bottom of the ditch, lowering the ground-water level by drainage will have no effect on the ditch. If a ground-water mound developed by seepage is in contact, and the water-table slope away from the ditch is steepened, then there will be increased seepage. Mr. Treadwell states:

> "Of course, all earthen ditches seep and the amount of the seepage is due to the porosity of the material through which they pass. Even a porous material is some support for the water in the ditch; otherwise all the water would seep from the ditch. The water that fills the pores of the material also acts as a support. If that support is taken away by a trench or sump near the ditch from which trench or sump water is pumped, then there would seem to be room for the claim that such artificial works induced seepage and thus took away the right of natural support. The writer can see no reason why this could not occur."

This is an excellent example of apparently good reasoning that overlooks the two fundamental hydraulic factors: (1) Whether or not the ground-water mound is in contact with the bottom of the stream; and, if so, (2) whether or not the slope of the mound is increased by the construction of the sump.

Mr. Treadwell states that

> "* * * he would like to read further evidence to support the positive rule announced by the authors that as long as there is a column of unsaturated material between the surface stream and the underground water table, a lowering of the water table will not affect the surface stream."

The writers would state that the existence of a water table below the bottom of an influent stream has been proved by measurements of depth to water table in test holes sunk in a stream bottom within the area covered by water. It has been found that measurements of stream loss (influent seepage) do not vary with distance between stream bottom and water table.

Mr. Treadwell suggests "* * * that the water table is at all times supplying this column with water, by capillarity, for several feet above the water table." A close study of Fig. 3 shows a subsurface "rainfall" generated at stream bottom and supplying the water table. The seepage column is a zone of descending ground water and is considered to be in the unsaturated zone because it is above the water table. However, probably most of the pore space is occupied by the influent seepage. Neither the water table nor capillarity furnishes water to this influent seepage column.

As is to be expected, Mr. Meinzer's discussion contains so many pertinent remarks with which the writers are in accord that there is no need to mention them here. He notes that the writers discuss effects on surface flow of pumping from influent and effluent subflow, but do not discuss in detail effects on subflow of diversions from influent and effluent surface streams. Although the principles governing these two cases are brought out in the paper, Mr. Meinzer's discussion of the importance of the effect of stream diversions on ground water corrects one of the defects in the presentation, caused by a desire for brevity.

Discussing the statement in the paper regarding development of cone of depression below stream bed, Mr. Lewis states: "This statement should be qualified to the extent of excluding those influent streams where the water table is not in contact with the surface stream."

It is interesting to note that, although the writers emphasized the importance of contact of ground water with stream bottom, they omitted the statement of this controlling feature in this one case, and they thank Mr. Lewis for bringing it to their attention. He also draws the contours of a ground-water mound due to stream flood, showing that the bend of contours takes place largely in the adjacent, less pervious material, rather than in the channel.

Mr. Snow's criticism has been read with particular interest because it presents the viewpoint of one versed in ground-water conditions in Eastern United States. The general ground-water equation, which evaluates rainfall, runoff, evaporation, and transpiration, can be applied better in the East than in the desert region where these factors are more variable. The study of ground-water supplies in pervious stream gravels incased in less pervious material is based chiefly on stream-flow measurements that give quantity of influent and effluent seepage.

Mr. Snow's interesting discussion of ground-water conditions in the East, and legal problems related thereto, are too numerous to admit of detailed discussion. His statement that the problem of pollution of ground water is generally more important than quantity of ground water applies to conditions in the humid East rather than in the semiarid West.

The discussion by Mr. Conkling is of value because his concept of a ground-water stream is much broader than that of the writers. He offers the following definition: "A stream is a body of water moving as a whole in an accustomed location and in a definite direction." Such a broad view would include deep artesian flow as a ground-water stream or any concentrated flow of ground water not connected or related to a surface stream.

This is a strictly engineering concept, and it would be necessary only to measure the rate and direction of ground-water movement to prove a subsurface stream. The writers' conceptions are geologic, and are based on the occurrence of ground water beneath the valleys and productive bottom lands of the major streams of Western United States.

This geologic concept is as follows: Many of the major streams of the West have broad alluvial valleys with marginal terraced lands several miles in width. These lands are underlain by pervious gravels deposited in a river-cut channel that has been excavated either in bedrock or in less pervious alluvial materials. Under these conditions, ground water moving in this pervious material may be properly classified as a subsurface stream which can be separated from the general body of percolating ground water beyond the outermost banks of the original channel, first excavated and later filled by the river.

It is only to occurrences such as this that the concept of a subsurface stream can be applied, and even in such cases difficulties may arise in separating the ground-water stream from the surrounding body of percolating water.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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# DISCUSSIONS

# THE GRAND CENTRAL TERMINAL IN PERSPECTIVE

## Discussion

By Messrs. Harold M. Lewis, and Bion J. Arnold

Harold M. Lewis,[30] M. Am. Soc. E. C. (by letter).[30a]—The transformation wrought in the vicinity of the Grand Central Terminal in the City of New York, as well as the incidental changes that have taken place in the suburban areas which it serves, have been described forcefully by Colonel Wilgus. These justify his calling the project "a successful adventure in civic planning." This discussion is limited to the general relationship of the project to the plan of the city and region within which it lies. From this point of view, it is of interest to consider what further improvements may be made in the approaches to the terminal, in either Manhattan or the Bronx, and how these might bring about improved development of adjacent territory. In addition, one would like to know to what extent other terminal areas might be transformed as a result of a shift from steam to electric operation.

The following two improvements to the approaches would have a far-reaching effect on abutting properties (see Fig. 16): (1) An elimination of all or part of the viaduct and elevated structure in Manhattan north of 96th Street; and (2) the development of an important sub-terminal in the vicinity of Mott Haven in the Bronx. Each of these has been mentioned by Colonel Wilgus.

The writer appreciates that any change in the New York Central Railroad tracks in Park Avenue would present a very difficult and costly problem for the railroad company, particularly if it should involve any change in the tracks now in the tunnel south of 96th Street. The study of the Regional Plan Association, Inc., referred to by Colonel Wilgus, proposed to leave these latter tracks, as well as those on the existing viaduct as far as 104th Street, unchanged in both grade and alinement, and involved track changes only north of 104th Street.

Note.—This paper by William J. Wilgus, Hon. M. Am. Soc. C. E., was published in October, 1940. *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1940, by Messrs. F. Lavis, E. R. Hill, Alonzo J. Hammond, and H. L. Ripley; January, 1941, by Messrs. A. J. Meehan, and J. P. Hallihan; and February, 1941, by Messrs. Arthur V. Sheridan, and C. E. Smith.

[30] Chf. Engr. and Planning Officer, Regional Plan Association, Inc., New York, N. Y.

[30a] Received by the Secretary January 10, 1941.

From that point, the tracks would descend at an easy grade into a tunnel at 116th Street, which would be facilitated by the fact that the grade of Park Avenue rises suddenly at this point. Between 116th and 122d streets this tunnel would swing through the blocks westward into Madison Avenue, which it would follow north of that point to a tunnel crossing of the Harlem River. In the Bronx, connections would be made to the Hudson River and Putnam divisions near McCombs Dam Bridge, to the Harlem Division near 163d Street and to the New York Central storage yard near 161st Street. The maximum grades involved on the main lines were 1.5% and about 1.7% on the connection to the storage yard.

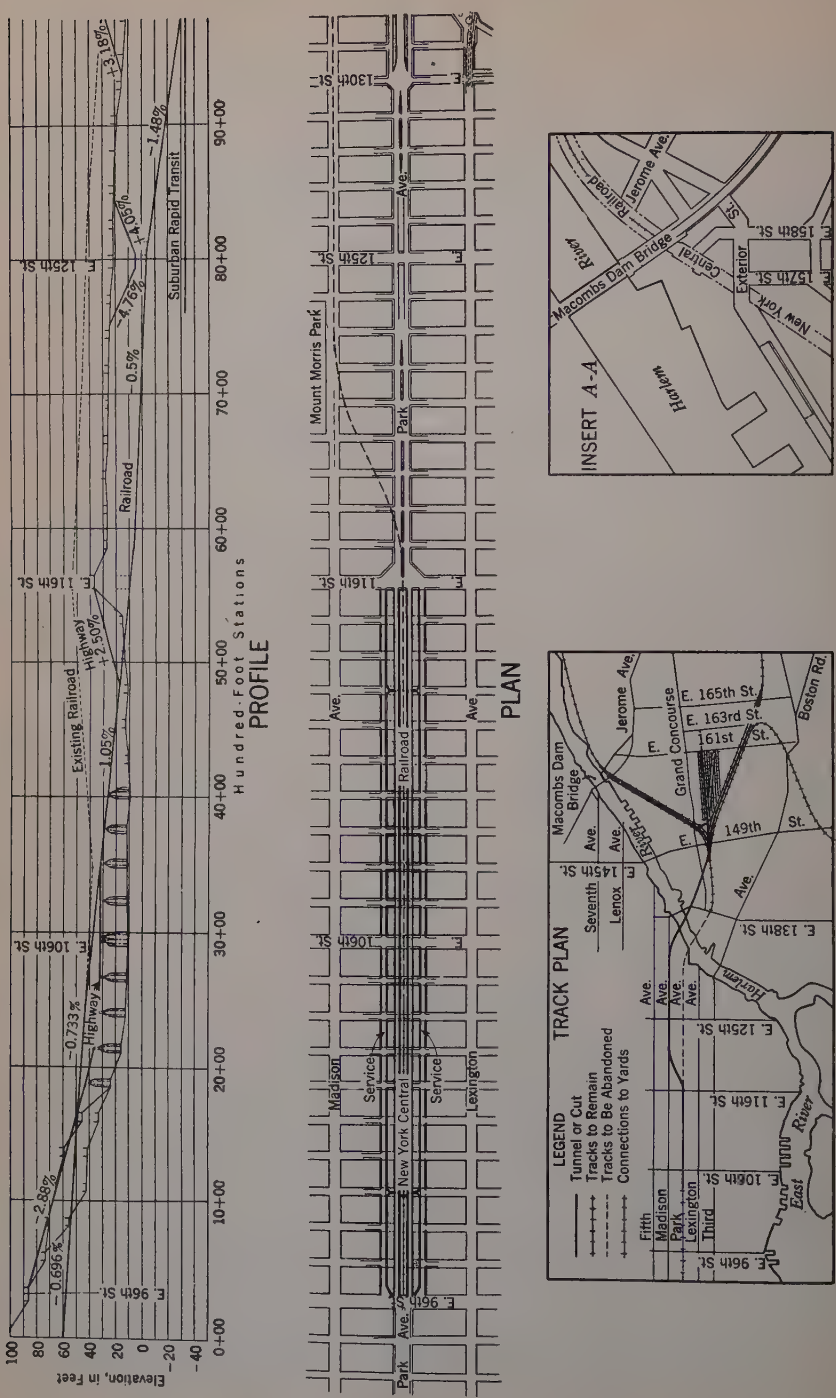
The three main objectives of this plan were: (1) To provide for an improvement of Upper Park Avenue and an improved highway connection between Manhattan and the Bronx; (2) to provide a connection with a possible future suburban transit line in Madison Avenue, as proposed many years ago by the Westchester County Transit Commission; and (3) to eliminate interference between rail and water movement at the present Harlem River Crossing of the New York Central Railroad.

The area on both sides of Park Avenue between 96th and 116th streets is a low-lying section, densely built up with tenements and seriously blighted by its proximity to the present railroad viaduct. North of 116th Street, adjoining property still suffers from the effect of the railroad (which is on an elevated structure north of 110th Street), but the land is about 15 ft higher in elevation, better developed and probably could be reclaimed following an elimination of the railroad structure.

Between 96th and 116th streets it was proposed to widen Park Avenue from its present width of 140 ft to 300 ft. On each side of the railroad tracks, and at about the proposed new level of these tracks, a three-lane, one-way express roadway would be constructed (see Fig. 17). Outside of this would be a landscaped slope and then a service road with a 30-ft roadway to serve abutting property. At 96th Street the express roadways would connect with the present roadways in Park Avenue south of that point; they would also meet the present street grade at 116th Street. Between those points there would be no grade crossings.

With the railroad eliminated, Park Avenue, north of 116th Street, could be developed as it has been south of 96th Street, except that a highway grade separation has been proposed at 125th Street. This might well be extended to include Park Avenue beneath 124th and 126th streets, both of which are important connections to the Triborough Bridge. A lift highway bridge across the Harlem River, with a clearance of 45 ft when closed, would provide a direct connection with Mott Avenue in the Bronx, widened in 1940 as a connection to the William F. Deegan Boulevard.

Other plans, developed by the city authorities, have proposed the widening of Park Avenue to 300 ft and the construction of an express highway all the way from 96th Street to the Harlem River with a high-level highway bridge over the river. These plans proposed no change in the railroad structure and their execution would make it almost impossible to ever make any change in the railroad tracks either within Manhattan or at the Harlem River crossing.


PROFILE
Hundred-Foot Stations
Elevation, in Feet
Existing Railroad
Highway
Railroad
Suburban Rapid Transit
E. 96th St.
E. 106th St.
E. 116th St.
E. 125th St.
PLAN
Mount Morris Park
New York Central Railroad
Service
Madison Ave.
Park Ave.
Lexington Ave.
INSERT A-A
Macombs Dam Bridge
Harlem River
Jerome Ave.
New York Central
Exterior St.
E. 157th St.
E. 158th St.
TRACK PLAN
LEGEND
Tunnel or Cut
Tracks to Remain
Tracks to Be Abandoned
Connections to Yards
Grand Concourse
E. 165th St.
E. 163rd St.
E. 161st St.
E. 149th St.
E. 138th St.
Boston Rd.
East River

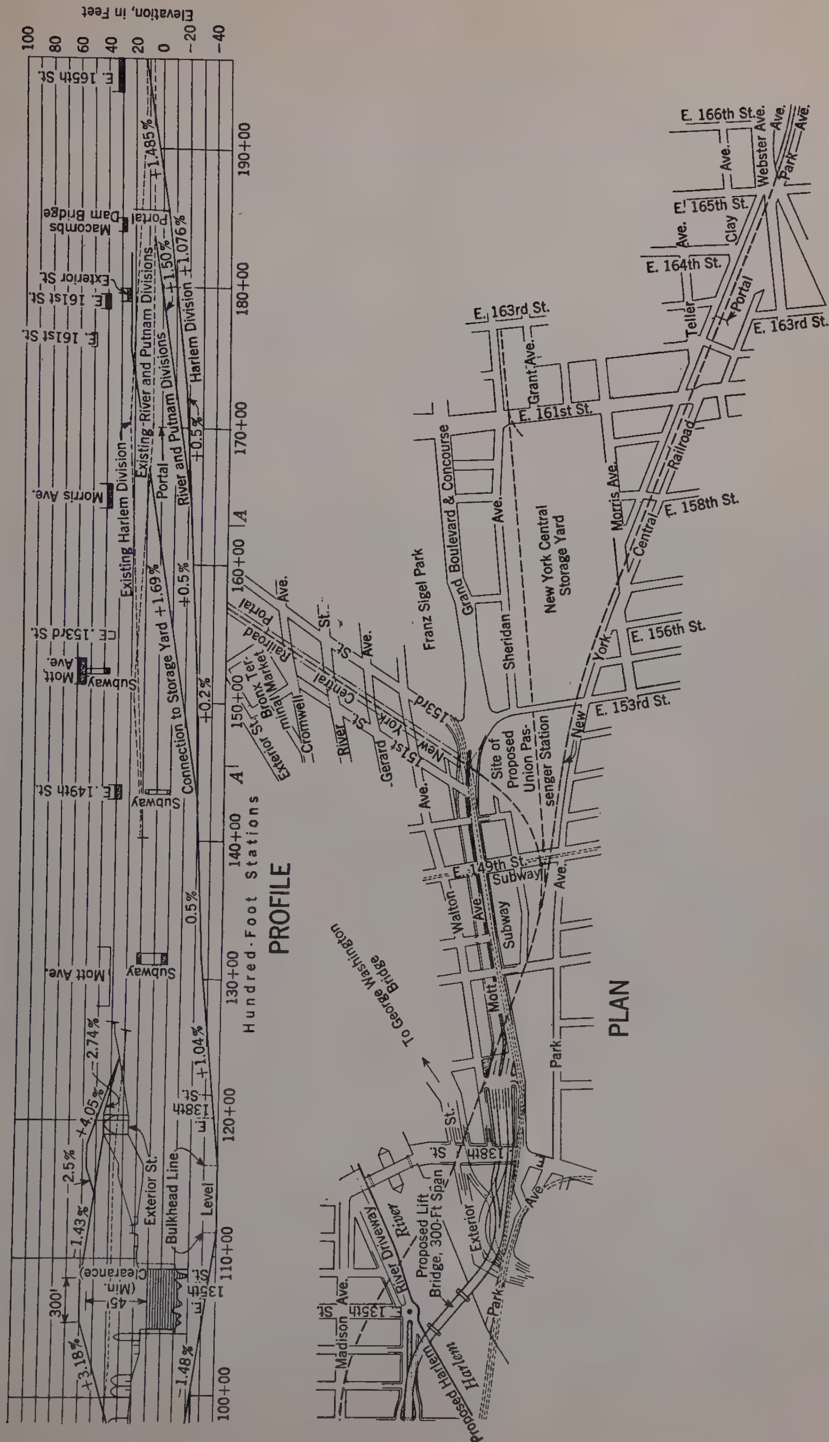



Fig 16.—Study for Improvement of Park Avenue, New York, N Y

Park Avenue can never be made an express street south of 96th Street, where it already carries a heavy load, and should not have to carry much additional traffic. For these reasons, the Regional Plan Association study led to the conclusion that a highway completely free from grade crossings all the way from 96th Street to the Bronx was not justified. It was not contemplated that the present high-class type of residential development along lower Park Avenue would continue north of 96th Street, but that the blocks on both sides of the widened Park Avenue between 96th and 116th streets could be utilized for low rental housing at a relatively low density. North of 116th Street the type of development now found around Mount Morris Park might be expected.

The proposed treatment of the railroad and highway could be carried out progressively, which is important if no immediate change in the railroad tracks is practicable. The first step would be the widening of Park Avenue between 96th and 116th streets and the construction of the proposed express roadways and service streets in this section. North of 116th Street, vehicles could be

FIG. 17.—PROPOSED TREATMENT OF PARK AVENUE, MANHATTAN, BETWEEN 96TH AND 116TH STREETS
(A Widened Right-of-Way Would Permit the Construction of Express Highways
On Each Side of the Railroad Right-of-Way)

accommodated partly on the surface of Park Avenue, beneath and alongside the railroad structure, and partly by diversion to Madison and Third avenues.

The railroad plan north of 116th Street could be carried out later without any interference with existing tracks and services, as both the line in Madison Avenue and the proposed Harlem River tunnels are far removed from any existing railroad structure. The railroad changes between 104th and 116th streets could be facilitated by diverting the existing tracks to temporary trestles

constructed in the landscaped areas between the express roadways and service streets, placing two tracks on each side. Some temporary changes would have to be made in the express roadways between 103d and 106th streets, reducing temporarily the clearance where 104th and 105th streets would pass beneath.

Following this temporary relocation of the tracks, the existing viaduct could be rebuilt at the proposed new grade, a connection made with the new tunnels north of 116th Street and the railroad viaduct removed north of that point.

A final step would be the construction of the proposed highway bridge over the Harlem River to connect Park Avenue with the Grand Boulevard and Concourse.

The Regional Plan contemplated the development of a series of important sub-centers within New York City as subsidiaries of the main business center, which it was expected would remain in midtown Manhattan. There is already a center of this type in Harlem, focused about 125th Street, but it was expected that another important business district would arise in the Mott Haven section of the Bronx. With that in mind, parts of the proposed regional highway and rail systems were shown as focused in this area where a union railroad passenger terminal was proposed. Colonel Wilgus has referred to such a terminal as a part of the plans of the New York Central Company that "has not been executed in full."

The study, by the Regional Plan Association, for the Park Avenue improvement showed this terminal just north of 149th Street and east of the new Bronx Post Office. It would be located beneath the present yard level and its platforms would serve eight tracks, connected through a mezzanine. Convenient connection could be provided with both the East Side and West Side subway systems in Manhattan.

From such a terminal, rapid bus connections could be provided to the Borough of Queens over the Triborough Bridge, easily reached by the William F. Deegan Boulevard. The Regional Plan also contemplated that when a suburban transit line from New Jersey is supplied over the George Washington Bridge, as provided for in the bridge design, this might be extended over the Harlem River to the Mott Haven Terminal, via a connection with the Hudson River Division of the New York Central Railroad.

Steam operation was terminated in 1931 in two other large yards of the New York Central System in New York City—the 30th Street and 60th Street yards on the West Side of Manhattan. It is probable that some of the air rights over these will be developed in future years, but there does not seem to be a likelihood of such sensational changes as took place over the old Grand Central Terminal yards. The Regional Plan did include some pictures showing the possibilities of continuing the apartment developments along Riverside Drive southward from 72d Street toward 57th Street, with the western terminus of a widened 59th Street as the center of a site for a major municipal sub-center, which might include a market, an arena, and other large buildings. This may never come, but, in any event, what has happened along Park Avenue has demonstrated that such changes would be perfectly feasible from an engineering point of view.

Bion J. Arnold,[31] M. Am. Soc. C. E. (by letter).[31a]—Being historical, based on facts and dates which the author has obviously been very careful to make correct, this paper leaves no chance for argument. Hence concurrence with statements in this excellent presentation seems to be the only course open; and especially is this so when one has had a part in compiling some of the data upon which final action was based.

As Colonel Wilgus states, it was the writer's lot to be called in August, 1901 (see heading "Grand Central Terminal Transformation—Formative Period from 1899 to 1907: Initial Move for Electrification"), "to study the feasibility of handling heavy through trains by electricity between Mott Haven and the terminal"—thus he was the first outside consultant upon whom the responsibility of securing definite conclusions and recommendations regarding the electrification was placed. Later he was a member of the Electric Traction Commission, which carried the responsibility of the actual designing and construction work. As he was paid a fixed fee for his investigation and report, he was at liberty to adopt any method he thought advisable to secure the necessary information upon which to base conclusions.

At that time, although there had been a number of railway electrifications of mixed types, there was no electrification approaching the magnitude of this one and no data that the writer could find, which could guide him in determining the amount of power required to propel heavy trains under conditions such as this installation involved. Colonel Wilgus, then chief engineer, made available all the facilities owned by the railroad company that were needed.

At that time there were few dynamometer cars in existence in the United States; but one car was owned jointly by the Illinois Central Railroad Company and the University of Illinois, at Urbana, Ill. In order to be certain that the car would be properly operated, the writer employed Edward C. Schmidt, Assoc. M. Am. Soc. C. E., then professor of railway mechanical engineering at the University of Illinois, and his staff. This car was coupled between the locomotive and the train on one of the heavy passenger trains operated between Chicago and New York and then used in the same way to measure the draw-bar pull of many trains running into and out of the Grand Central Terminal in New York, requiring several weeks.

From the records thus secured the draw-bar pull required to accelerate and propel each class of train at various speeds was obtained and when reduced to a horsepower basis and applied to all trains (more than 600 daily) that ran over the division, the total maximum power was determined, as well as the average power required to duplicate the train service then in operation between Mott Haven and the Grand Central Terminal.

The next problem was to ascertain what increased rate of acceleration with the different trains could be obtained by electrical operation and the consequent increase in the power required for such operation. To meet this condition arrangements were made with the General Electric Company to furnish the use of two motor cars, which could be used in a series of tests to propel the train

[31] Cons. Engr., Chicago, Ill. (Col., Air Corps, Inactive Reserve, U. S. Army).

[31a] Received by the Secretary February 7, 1941.

furnished by the railroad company on the General Electric Company's test tracks at Schenectady, N. Y., in comparison with the same train operated by the most powerful steam locomotive designed especially for suburban service that the railroad company then owned and having practically the same weight on its drivers as the two motor cars had combined. The electric tests were made on the General Electric test track, located on the berm bank of the old canal at Schenectady; but the curve leading from the New York Central tracks into this track was too sharp for the locomotive to go around it, so that the tests of the steam locomotive and the trains were made on the west-bound freight track of the main line west of Schenectady, proper adjustments being made for grade, curvature, and wind resistance on the different runs. These tests, and the necessary mathematical calculations and deductions from them to reach conclusions, involved a vast amount of work by many men.[32]

In determining the form and best method of producing and applying the electricity necessary to propel the trains, twelve different types and locations of power stations and methods of distribution were outlined and their relative efficiencies estimated. In general, the results of these tests and analyses showed, and the writer's recommendations were to the effect, that the electrification of the terminal between Mott Haven and the Grand Central Station was feasible. The saving in operation effected thereby was only slightly in favor of electrification when the fixed charges, made necessary by the installation, were taken into account. Therefore the installation could not be justified because of these economies of operation alone; but it could be justified on other grounds such as safety, increased capacity, etc., or by the extension of the electrified zone. This conclusion has been thoroughly demonstrated as sound by the results of many years of successful and profitable operation, as testified by Colonel Wilgus.

When engaged on the work of this terminal the writer had the good fortune to occupy joint offices with Charles A. Reed, the constructive genius of Reed and Stem, the architects who, working as a firm with the railroad officials, prepared the first plan (see Fig. 9) showing a twenty story station with its accompanying court of honor. This plan of utilizing "air rights" so impressed the writer that when later acting as consultant for the City of Chicago, Ill., on railway terminals, after months of strenuous argument, he saw his own ideas in this respect prevail. As a result the vast area then owned by the railroads contiguous to, and west of, the Chicago River began to be dotted with "air right" structures. Prominent among these structures are the new U. S. Post Office, the Pennsylvania Railroad Freight Station, and the Chicago Daily News Building, forming the nucleus, if electrification ever comes, of another Grand Central Development of "air right" structures.

Later the same "air right" ideas prevailed in the minds of those who developed the railway station at Cleveland, Ohio, and the writer is of the opinion that the surplus strength in the footings and heavy columns which Mr. Reed

[32] "Method of Ascertaining by Means of a Dynamometer Car the Power Required to Operate the Trains of the New York Central & Hudson River Railroad Between Mott Haven Junction and Grand Central Station, and the Relative Cost of Operation by Steam and Electricity," by Bion J. Arnold, *Transactions*, Am. Inst. of Electrical Engrs., Vol. XIX, 1903, p. 865; also "Comparative Acceleration Tests with Steam Locomotive and Electric Motor Cars," by W. B. Potter and Bion J. Arnold, *loc. cit.*, p. 833.

and Colonel Wilgus succeeded in embodying in the present Grand Central Station Building in New York (but now lying dormant) will yet be utilized by the extension of the station building upward. Thus another step will be completed, as conceived in the original plan, which like several other steps mentioned in the paper, although delayed, have shown their merit and been completed since.

The author of the paper is to be commended for his thoroughness in compiling the vast amount of historical and engineering data, and the Society is to be congratulated upon having this matter spread upon its records in such satisfactory sequence and form.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# THEORY OF ELASTIC STABILITY APPLIED TO STRUCTURAL DESIGN

## Discussion

BY MESSRS. HAROLD D. HUSSEY, H. N. HILL, AND F. H. FRANKLAND

HAROLD D. HUSSEY,[13] M. AM. SOC. C. E. (by letter).[13a]—The "Bibliography" emphasizes the fact that this subject has received very little discussion in the United States. Of the fifty-two items listed, only two were published in this country—one book and one paper. (The latter, (33),[13b] is a short, concise discussion of several practical problems and deserves study by structural engineers.) Four others were published abroad in English. The remaining forty-six items are in foreign languages.

The paper is welcomed as a contribution within easy reach of American engineers. It confirms present practice with respect to problems in buckling of plates as found in standard specifications, with modifications as noted herein.

The general expression for elastic stability of steel plates subject to compression or shear, as given by the authors in Eq. 1, can be expressed (assuming $E = 29{,}000{,}000$ and $\frac{1}{m} = 0.30$) as:

$$S_{cr} = 26{,}200{,}000\, k\, \zeta \left(\frac{t}{d}\right)^2 \qquad (53)$$

in which $S_{cr}$ = critical compression or shear. Solving for $\frac{d}{t}$,

$$\frac{d}{t} = 5{,}120 \sqrt{\frac{k\,\zeta}{S_{cr}}} \qquad (54)$$

If $S_{cr}$ is assumed to be equal to the actual stress $S$ times a factor of safety, and

NOTE.—This paper by Leon S. Moisseiff and Frederick Lienhard, Members, Am. Soc. C. E., was published in January, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: November, 1940, by Louis Balog, Esq.; and February, 1941, by Joseph S. Newell, Esq.

[13] Designing Engr., Am. Bridge Co., New York, N. Y.

[13a] Received by the Secretary January 27, 1941.

[13b] Numerals in parentheses, thus: (33), refer to corresponding items in the Bibliography in the Appendix of the paper.

$k$ and $\zeta$ are evaluated, the thickness ratio can be expressed in the general form

$$\frac{d}{t} = \frac{\text{coefficient}}{\sqrt{S}} \qquad (55)$$

The authors have given the values of $k$ and $\zeta$ for plates subject to compression and shear.

Table 4 contains values of $\frac{d}{t}$ for column plates and outstanding flanges. Under the heading "Vertical Stiffeners of Webs of Plate Girders: Spacing of Vertical Stiffeners," proposed values of the coefficient in Eq. 55 are given for

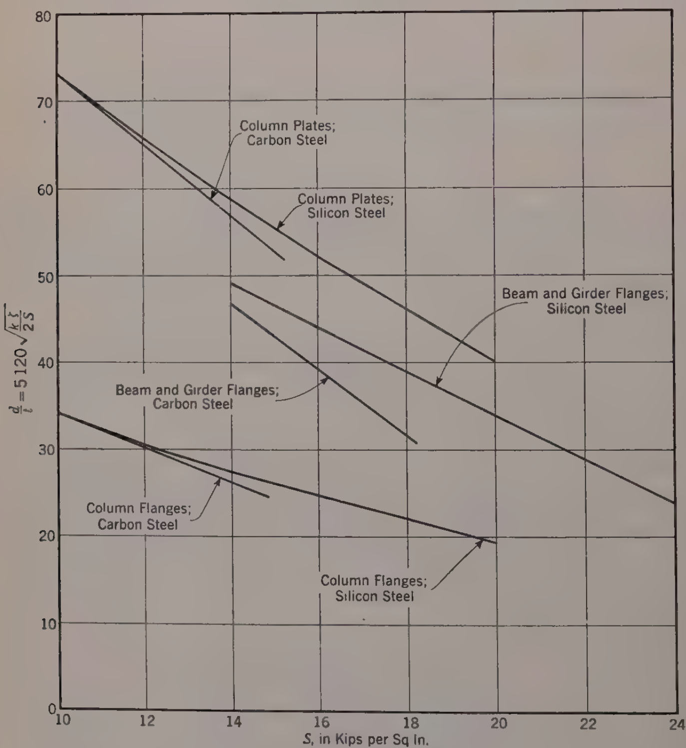



Fig. 11.—Allowable $\frac{d}{t}$ Ratios

intermediate stiffener spacing and for girders requiring no intermediate stiffeners.

Values of $\frac{d}{t}$ for outstanding parts of columns, as given in Table 4(*b*), are open to question. The minimum value of $k$ (this assumes that the outstanding part is free to rotate) was used in computing Table 4(*b*) in order to allow for initial curvature. This is equivalent to an assumption that the column has no torsional rigidity, which is not true in a practical case. It would seem more

reasonable to assume an elastically built-in edge and a value of $k$ equal to twice the value assumed.[14] This would increase the values of $\frac{d}{t}$ given in Table (4$b$) by 41.4%.

The width-to-thickness ratios given in Table 4 show an important characteristic when examined in the light of current specifications. The latter fix $\frac{d}{t}$ ratios for maximum allowable unit stresses and, for smaller stresses, provide that $\frac{d}{t}$ may be increased in proportion to the square root of the ratio of allowable to actual stress.[15] The values of $\frac{d}{t}$ in Table 4, when plotted in Fig. 11, show an approximately uniform variation. The same is true of values of $\frac{d}{t}$ for depth-to-thickness ratios for girder webs, plotted in Fig. 12. This characteristic is due to the influence of the "modulus factor" $\zeta$. A study of these curves shows that, for stresses less than the allowable, $\frac{d}{t}$ should be increased in direct proportion to the ratio of allowable to actual stress.

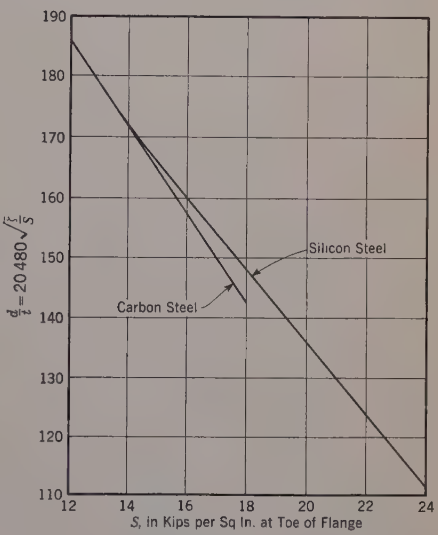



FIG. 12.—ALLOWABLE CLEAR DEPTH-TO-THICKNESS RATIOS FOR GIRDER WEBS

The authors have developed web slenderness ratios of 165 and 136 for girder webs of carbon and silicon steels, respectively (see heading "Chapter 3.—Design of Webs for Plate Girders"). These values compare with 170 and 145 used in current specifications.[16] The authors have used the clear distance between the flange angles as the assumed depth of web. A value of 24 for the buckling coefficient $k$, assumed by them, is for a plate simply supported on all sides (that is, free to rotate). The flange angles will provide some restraint to the free rotation of the web. In deriving the current values of web slenderness, it was assumed that this restraint extended a distance of five times the thickness of the web away from the flange.[17] This justifies the higher values in current specifications.

The authors have developed a new formula for the spacing of intermediate stiffeners of plate girders (Eq. 35$a$). Because of its simplicity and a greater accuracy, it is recommended as a substitute for the formulas in current use.

[14] "Theory of Elastic Stability," by S. Timoshenko, First Edition, McGraw-Hill Book Co., Inc., New York and London, 1936, p. 342.

[15] A. R. E. A. Specification for Steel Railway Bridges, 1938, paragraphs 405 and 431.

[16] *Loc. cit.*, paragraphs 431 and 1602.

[17] "Elastic Stability of Plates Subjected to Compression and Shear," by O. E. Hovey, Hon. M. Am. Soc. C. E., *Proceedings*, A. R. E. A., Vol. 36, 1935, p. 721.

H. N. Hill,[18] Assoc. M. Am. Soc. C. E. (by letter).[18a]—The problem dealt with in this paper is one that should command the increasing attention of structural engineers. The problem consists of expressing the solutions for various instability problems connected with structural design in such a form that they may be readily understood and conveniently applied by the designing engineer. The paper is of particular value since it treats a subject which, for many purposes, is inadequately covered in current specifications for structural steel design. With the exception of the column formulas, the rules in current structural steel specifications covering features of design controlled by stability considerations have been formulated to obtain extreme simplicity, at the expense of accuracy.

The authors have attacked the problem of deriving more rational methods for the design of stiffened and unstiffened column plates and plate girder webs, by application of solutions obtained from the theory of elastic stability. The reader cannot fail to be impressed with the great amount of work that must have been involved in the preparation of such a paper. In the derivation of design formulas from complicated general solutions of stability problems, certain simplifying assumptions must necessarily be made. The value of the resulting formulas will depend on the extent to which the simplifying assumptions can be justified. As the authors state, experimental verification is needed to encourage the use of design methods thus obtained. This is particularly true in the case of design formulas, such as Eq. 31, the theoretical derivation of which is not truly rigorous in all its aspects.

The remarks that the writer has to make concerning this paper are offered in the hope of clarifying certain assumptions made by the authors and, perhaps, of contributing to the completeness of the subject.

The necessity for basing design involving stability considerations on a more comprehensive treatment of the problems involved than that embodied in the simple rules of the standard structural steel specifications becomes of increasing importance when designing with high-strength alloy steels or high-strength aluminum alloys with their relatively high ratio of strength to modulus of elasticity. In such structures light weight and consequent economy of material is frequently the primary consideration. Design in the aircraft industry probably represents the extreme case in which a high strength-to-weight ratio is the primary objective. It is in connection with this industry that the greatest advances have been made in methods of structural design involving considerations of stability. Perhaps the extensive bibliography which the authors have included might be augmented by reference to the numerous publications of the National Advisory Committee for Aeronautics on stability problems in general and stiffened flat plates in particular. Many of these publications are translations of papers that appeared originally in foreign languages.

In connection with Table 1, a "partly clamped" edge condition is assumed for the case of shear in panels having length-to-width ratios of less than 0.5. Since the length-to-width ratio $\beta$ represents the proportions of a plate, a plate having a $\beta$-value less than one half can be expressed as having a $\beta$-value

[18] Research Structural Engr., Aluminum Co. of America, New Kensington, Pa.

[18a] Received by the Secretary February 4, 1941.

greater than 2 simply by transposing the length and width terms. The justification for assuming partial edge fixity for plates having $\beta$-values less than 0.5 is not obvious. This is an assumption that may require experimental verification.

In Table 2, $k$-values are given that are applicable to plates subjected to combined bending and shear. It should be remembered, however, that these $k$-values are not applicable to plates subjected to other combinations of stress. Similar values are available for the case of a plate with simply supported edges subjected to combined uniform compression in one direction and shear (11). Fig. 13 shows a curve representing these values compared with a similar curve

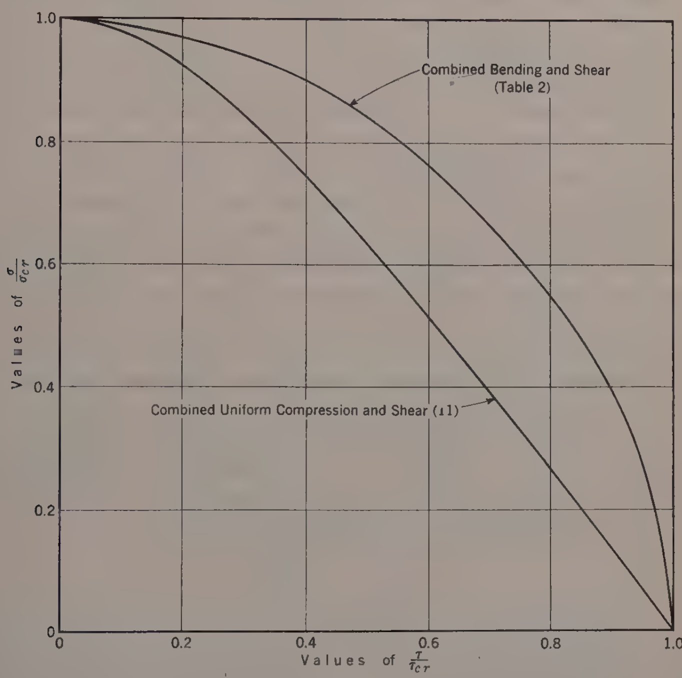



FIG. 13.—CRITICAL STRESSES FOR A SIMPLY SUPPORTED FLAT PLATE SIMULTANEOUSLY SUBJECTED TO DIRECT STRESS AND SHEAR

for the values given in Table 2. They indicate that the presence of the uniform compressive stress produces a greater reduction in the critical shear stress than is the case for combined bending and shear.

The authors state that for stresses beyond the yield strength of the material, the modulus factor $\xi$ is theoretically zero. This is undoubtedly true for mild steel that has a definite yield point. The statement does not apply, however, to those materials that have no definite point of yielding. Experience with the aluminum alloy cited by the authors (27S-T) indicates that the modulus factor at the yield strength (stress corresponding to 0.2% set) is generally about 0.4.

In Table 3 the authors give stresses and corresponding modulus-factor values, from which the critical stress can be determined by a trial-and-error procedure. In problems involving buckling at stresses in the plastic range, the writer has found it convenient to express the relation between stress and reduced modulus in the form of a curve in which stress is plotted against the ratio of stress to corresponding reduced modulus value. The formula for critical stress (Eq. 1) can be rewritten by transposing $E$ to the left-hand side. The values for the term $\frac{\sigma_{cr}}{E}$ can then be calculated from such an equation, and corresponding critical stress values can be picked directly from the curve.

Eq. 21 has been obtained by expressing $k_e$ and $\Sigma \sin^2 (\pi C_s)$ in terms of the number of stiffeners ($N$) so as to satisfy the values given in Table 5. The resulting equation is correct for values of $N = 1, 2,$ or 3. This formula (Eq. 21) could be made generally applicable for any number of stiffeners by substituting the expression $4(N + 1)^3$ for the coefficient of $\delta_s$ in the numerator. Similarly, Eq. 22 can be made to agree with Eq. 20 for any value of $N$ by replacing the coefficient for $\delta_s$ by the expression $2(N + 1)^3$.

According to Eqs. 35 and 36, the maximum spacing of intermediate stiffeners is limited to about 1.1 times the clear depth of the web. This limitation results from the fact that in arriving at a constant value for the coefficient in Eq. 34, the authors have considered only shear panels having $\beta$-values $\leqq 1$. If $\beta$-values greater than 1 are considered, the value of the coefficient is found to increase quite rapidly, and the use of a constant value is no longer satisfactory.

A simple expression for the required spacing of intermediate vertical stiffeners, which involves both the shear stress and the web slenderness $\frac{d}{t}$ and which is not limited to a narrow range of $\beta$-values, may be derived as follows: For the case of shear in plates having simply supported edges, the $k$-value of Eq. 1 can be very closely approximated by the equation,[19]

$$k = 5.35 + 4\left(\frac{d}{L_s}\right)^2 \qquad \text{(56)}$$

Substituting this value in Eq. 1, using a factor of safety of $1\frac{1}{2}$ on the shear strength and simplifying, yields for the required spacing of stiffeners the equation,

$$L_s = \frac{0.865}{\sqrt{0.341\,\dfrac{1 - \left(\dfrac{1}{m}\right)^2}{E}\,\tau - \left(\dfrac{t}{d}\right)^2}} \qquad \text{(57)}$$

which can be reduced to the following equations for steel and aluminum:
For steel,

$$L_s = \frac{865}{\sqrt{0.0107\,\tau - 1{,}000{,}000\left(\dfrac{t}{d}\right)^2}} \qquad \text{(58}a\text{)}$$

[19] See "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., 1936, p. 361.

for aluminum,

$$L_s = \frac{865}{\sqrt{0.0295\,\tau - 1{,}000{,}000\left(\frac{t}{d}\right)^2}} \quad \text{(58b)}$$

The paper deals almost entirely with the design of column plates and plate girder webs. Brief mention is made, however, of the buckling behavior of compression flanges. Space limitations probably prevented the authors from dwelling longer on this particular stability problem. As the authors state (following Eqs. 11), the buckling behavior of compression flanges between lateral supports is vastly different from that of a column subjected to axial stress, because the tension part of the beam has a stabilizing influence on the buckling of the compression flange. This stabilizing influence involves the resistance of the beam to twisting. Design formulas which do not take into account the torsional stiffness of the beam should not be expected to provide an adequate treatment of the problem. After numerous efforts to devise a simple yet adequate design formula, the writer has come to the conclusion that if such a formula is to be generally applicable to all symmetrical or nearly symmetrical flanged beams with single webs, the equation derived from a theoretical treatment of the problem represents the simplest practical form.

As a basis for deriving a general design formula for allowable stresses in compression flanges of beams, it seems logical to assume that the unsupported length is subjected to pure bending and that the ends are free to rotate about an axis in the web normal to the neutral axis of the beam (that is, the ends are not restrained against lateral bending). The critical stress for a symmetrical **I**-beam under these conditions can be expressed[20] as

$$\sigma_{cr} = \pi \frac{\sqrt{E\,I_1\,J\,G}}{S\,L} \sqrt{1 + \frac{\pi^2\,E\,I_f\,h^2}{2\,G\,J\,L^2}} \quad \text{(59)}$$

in which $I_1$ = moment of inertia about the principal axis in the web (in.$^4$); $J$ = section factor for torsion (in.$^4$); $L$ = laterally unsupported length (in.); $I_f$ = lateral moment of inertia of one flange (in.$^4$); $h$ = depth of beam (in.); $S$ = section modulus about the principal axis normal to the web (in.$^3$); and $G$ = modulus of elasticity in shear:

$$G = \frac{E}{2\left(1 + \frac{1}{m}\right)} \quad \text{(60)}$$

Although Eq. 59 was derived for symmetrical **I**-beams, it is also a very close approximation for channel sections and nearly symmetrical **I**-sections, if the value used for $I_f$ is that for the compression flange. In the case of symmetrical **I**-sections, $I_f$ can be taken as one half $I_1$. Eq. (59) can be generalized

[20] "Theory of Elastic Stability," by S. Timoshenko, McGraw-Hill Book Co., Inc., 1936, p. 261.

so as to cover buckling beyond the elastic range, and written

$$\frac{\sigma_{cr}}{E_x} = \frac{\pi}{\sqrt{2\left(1+\frac{1}{m}\right)}} L^2 S \sqrt{I_1\left[J L^2 + \pi^2\left(1+\frac{1}{m}\right) I_f h^2\right]} \quad \ldots .(61)$$

in which $E_x$ is the modulus of elasticity corresponding to $\sigma_{cr}$. By introducing a factor of safety, Eq. 61 can be reduced to a design formula, for any given material. If desired, the effect of lateral end restraint can be introduced by replacing the unsupported length $L$ in Eq. 61 by an equivalent length $K\,L$, as is frequently done in dealing with columns. For unrestrained ends, $K = 1$, and for completely restrained ends, $K = 0.5$. Further refinement could also be obtained by introducing a coefficient to take account of different loading conditions. Values for such coefficients have been published for numerous loading conditions.[21]

Values for the torsion factor $J$, used in Eqs. 59 and 61, are given in some handbooks[22] for various sizes of I-beams and channels. A reasonably close approximation for this value may be obtained for any single web section by considering the cross section as composed of a series of rectangles. The torsion factor for the section is then the sum of the torsion factors of the various elements:

$$J = \frac{1}{3} \Sigma\, b\, t^3 \quad \ldots \ldots \ldots \ldots \ldots \ldots .(62)$$

in which $b$ is the length of the rectangle and $t$ is the width.

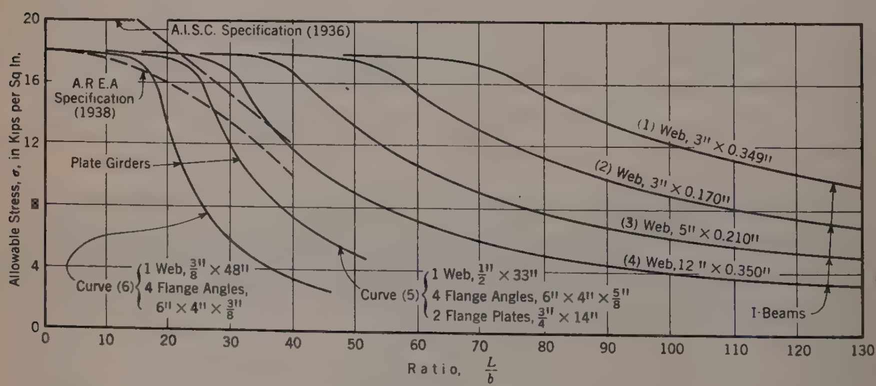



FIG. 14.—ALLOWABLE STRESSES FOR LATERALLY UNSUPPORTED FLANGES OF STEEL BEAMS WITH I-SECTION

The hopelessness of trying to obtain an adequate design formula expressing the allowable stress as a function of the ratio $\frac{L}{b}$ of the compression flange is demonstrated in Fig. 14. Curves in this figure indicate the relation between

[21] "The Lateral Instability of Deep Rectangular Beams," by C. Dumont and H. N. Hill, *Technical Note No. 601*, National Advisory Committee for Aeronautics (N. A. C. A.).

[22] Bethlehem Manual and Structural Aluminum Handbook.

allowable stress and unsupported length for mild steel beams of various I-sections, including standard I-beams and plate girders. These curves represent values for allowable stress calculated according to Eq. 61, assuming the material to have a yield point of 36,000 lb per sq in., and an elastic limit of 32,000 lb per sq in., and based on a factor of safety of 2. The allowable stress for a compression flange, as indicated by the American Railway Engineering Association and American Institute of Steel Construction specifications, is also shown in the figure. It is evident from a comparison of the curves in Fig. 14 that the specification formulas give values for allowable stress that are ultraconservative in the case of small I-beams, and that provide a factor of safety smaller than that intended in the case of deep plate girders.

The formulas given in current specifications for the allowable stress in compression flanges apply to all built-up members subjected to bending. This classification includes double web or box girders. The lateral stability of a box girder is much greater than that of a single web girder having the same flange width. This is obvious from a consideration of the greater lateral bending stiffness and torsional stiffness of the box section as compared with an I-section. It can be demonstrated[21] that, depending on the ratio of width to depth, the critical stress for a thin-wall, deep rectangular tube may be as much as three times as great as the critical stress for a solid rectangular beam of the same dimensions, loaded and supported in the same manner. In general, the lateral bending stiffness, and particularly the torsional stiffness of box sections, are relatively so great as to make a consideration of the possibility of lateral buckling of such a beam unnecessary.

F. H. Frankland,[23] M. Am. Soc. C. E. (by letter).[23a]—A study of this paper can lead only to the conclusion that its contents will have a far-reaching effect in the establishment of improved and more intelligent design methods as applying particularly to plate-girder and cellular-plate constructions.

Recent investigations into the stress distribution under load of such constructions have indicated clearly that many engineers are dissatisfied with the pertinent so-called standard design methods of the past, which, to a great extent, ignored the fundamental laws of structural behavior.

This paper is a contribution of great importance to the profession in that it sets forth in a clear and understandable manner the general characteristics of buckling within the zone of elastic stability. This is of basic importance in the establishment of improved design methods whereby engineers may construct safer, more efficient, and more economical structures.

Engineers are experiencing a steadily changing relationship between the cost of labor and the cost of materials in structures, so that what may have been economic methods of design a few years ago are not necessarily economic today or tomorrow. This economic change is proceeding at an increasing tempo, and it therefore behooves the structural designer to improve his design methods by studying the results of such valuable investigations and studies as are embodied in this paper. The simplicity of the old design conceptions as

[23] Director of Eng., Am. Inst. of Steel Constr., Inc., New York, N. Y.

[23a] Received by the Secretary February 17, 1941.

applying to the elastic stability of structures, subjected to compressive stresses, no doubt relieved many designers from intellectual effort; and therefore these old conceptions are popular with some who may be expected to resist progress.

The aroused interest, due to the increasing preference for plate instead of truss construction evident during the past few years, in the elastic stability of plates, and in the stress distribution in plate structures, will be aided greatly by the information presented in this paper, as will the various investigations, now current, into different phases of the subject.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# RECOMMENDED PRACTICE AND STANDARD SPECIFICATIONS FOR CONCRETE AND REINFORCED CONCRETE

## Discussion

By Jacob Feld, M. Am. Soc. C. E.

Jacob Feld,[55] M. Am. Soc. C. E. (by letter).[55a]—This latest Report of a Joint Committee compares favorably with the previous reports and provides a much needed standard in concrete design as well as in concrete construction procedure. Just the statement that this Report answers many questions noted in the office copy of the 1924 Report—questions that arose in various design problems—shows the completeness of the Committee's work. However, the art of concrete construction as well as the science of concrete design will continue to develop, and further revisions of the proposed specifications and recommended practice must be expected.

There are some items which might be made more definite or which require change to conform to standard practice. The discussion of such items is grouped to conform to the divisions in the Report.

*Chapter I—Scope and Definitions.*—The definition of "laitance" should not be restricted to the material resulting from the use of excess mixing water, since similar material can be brought to the surface by too much troweling in any mix. It should be emphasized that the "modular ratio" is the ratio of the moduli of elasticity in compression. A clearer definition of "plastic flow" is the change of shape without change in volume; the suggested wording "inelastic deformation" indicates failure. There seems to be little reason for the "ratio of reinforcement" being a function of the effective area of the concrete rather than the total area, for the two areas are identical in columns, even in spirally reinforced columns, and there is considerable question as to the value of the effective area of a T-beam. The definition of "saturated and surface dry" is far from clear or definite; it might be better to define that condition of the aggregate as the result of air-drying a saturated sample.

Note.—This Report was published in June, 1940, *Proceedings*, Part 2. Discussion on this Report has appeared in *Proceedings*, as follows: September, 1940, by L. J. Mensch, M. Am. Soc. C. E.; November, 1940, by Messrs. John C. Sprague, and Walter R. Hnot; December, 1940, by Edward C. Gould, Assoc. M. Am. Soc. C. E.; and February, 1941, by O. G. Julian, M. Am. Soc. C. E.

[55] Cons. Engr., New York, N. Y.

[55a] Received by the Secretary February 3, 1941.

*Chapter II—Materials.*—The writer agrees fully with the recommendations concerning the use of admixtures, and also would like to see the inclusion of a suggestion that the use of different brands of cement be investigated to give better workability. Actual experience with several brands of cement in the New York area, with identical water and aggregate ratios, shows some remarkable variation in strength as well as workability. In this group of cements (all "standard" portland, A. S. T. M. Specification C 9–38, with identical aggregates) the necessary water to give the same slump varies between 33 and 37 gal per bag of cement. The effect of such variation on a strict application of the water-cement ratio method of concrete mix design can be imagined.

211—Aggregates—Fireproofing.—The classification of less fireproofing value for certain aggregates is justified, not because they "change in volume a relatively large amount," but because of unequal expansion coefficients along different axes. The tendency for some specification writers to classify the two groups of aggregates loosely as "non-silicic" (Group 1) and "silicic" (Group 2) should be discouraged. These terms appeared in a specification for a municipal structure, and the writer was asked by a bidding contractor to define "non-silicic concrete." Literally, such a concrete must be free of silica-bearing rock, gravel, and sand, as well as of cement.

A strength specification for coarse aggregate should also be included. The writer has in mind one source of gravel, in common use, which produces concrete that has never been known to show compressive strengths of more than 3,300 lb in standard cylinders 28 days old, no matter what kind of, or how much, cement is used in the mix.

216—Recommended Sizes.—A simplification of the recommended standard sizes of rods (R26–30) was much needed in 1930. However, it has always seemed to the writer that the ½-in. square rod was far less useful than one between the ⅜-in. round and the ½-in. round, having an area of 0.15 sq in. The series of standard rods would then have the areas substantially in the ratio of 5, 10, 15, 20, 30, 45, 60, 80, 100, 125, and 150 instead of the present standard series of 5, 10, 20, 25, 30, 45, 60, etc. The argument that the ½-in. square rod has greater bond surface per unit area than the round rods has little value in fact. An examination of present types of deformed bars shows no difference in shape or surface between square and round rods.

208–S—Grading.— Although a large range in sand gradation may be justified in concrete mix designs, resulting from careful tests of various mixes, a more rigid requirement must be set for the large amount of "average concrete," usually specified definitely by ratios. The writer would suggest that the specified sand be near the finest grading allowed in 208–S, since a surplus of fines in concrete will do much less harm, if any, than would a shortage of fines.

*Chapter III—Proportioning, Mixing, Curing, and Testing Concrete.*—

302—Basis for Specification.—Under Alternate A, many engineers do not permit less than certain minimum cement contents in the required concrete mixes, and they should specify definitely what minimums must be provided.

Table 1 is apparently based on normal portland cement; the values must be changed for special cements as well as for standard cements known to act

abnormally. In Table 3 it should be made clear that Col. 5 is the percentage of fine aggregate to the total of separate coarse and fine aggregates, volumes, or weights, not to the total mixture of all aggregates.

302–SA—Determination of Proportions.—The determination of a water-cement ratio for the desired strength of concrete should be from the test results, but must not be extrapolated beyond the actual test results. The required concrete strength for design purposes should be exceeded by 15% in the standard test specimens made in the laboratory. Actually, there is little relationship between the strength of a 6-in. by 12-in. test cylinder and the actual concrete in flexural compression in a slab or beam. The excess strength is merely the known surplus to be expected in laboratory concrete as against the average concrete in the field.

The writer is in full agreement with the elimination of any formula that purports to prophesy the 28-day strength of concrete cylinders based on 7-day tests. A little laboratory experience will convince any one of the possible variations in such a formula, even with all materials unchanged. Different brands of cement, all other factors remaining constant, will cause more than 100% variation in the ratio of 28-day to 7-day strengths.

The rather high initial cost of a proper set of tests for mix proportioning makes Alternate A uneconomical for contracts involving less than 10,000 cu yd of concrete. To take advantage of the savings from this alternate, central concrete plants should be permitted to furnish smaller volumes, based on test data of known aggregate sources and cement brands.

306–SA—Changes in Proportions or Materials by the Engineer.—The engineer should be permitted to order changes in proportions or materials if either the workability or the strength is not satisfactory.

302–SB—Cement Factor.—Without a special heading, the Report gives a definition of a "cubic yard of concrete." It might be well to make clear that this volume of freshly mixed concrete will not always become a cubic yard after setting up.

312–S—Machine Mixing (at Site or at Central Mixing Plant).—The mixing times specified are not long enough to obtain a uniform mix in the larger drums. The mixing times given are:

| Capacity of drum, in cu yd | Mixing time, in min |
|---|---|
| 4 | 2.5 |
| 5 | 3 |
| 7 | 4 |
| 9 | 5 |

The chief difficulty with such short mixing periods is the loss in time when the cover is unlatched after mixing. The concrete thus being found unsatisfactory, more water is added and the mixing must be continued after replacing the cover. The writer would recommend that at least 2 min be added to each period; the formula then becomes 2 min per cu yd plus 1 min.

The Committee's recommendations for curing the concrete to meet strength requirements, when the 28-day specimens are below standard, answer the most

serious objection to the use of Alternate A. The monetary penalty (the additional curing costs) seems to be too low. Possibly the type of penalty imposed by the Highway Departments in the City of New York—that is, proportional payment for concrete subbase in direct ratio to the strength of 28-day cores—might be considered. Some such solution will be desirable to avoid a lawsuit if the additional curing does not provide the required strength in a reasonable time. The Committee makes no recommendations for correcting below-standard concrete in Alternate B.

*Chapter IV—Forms and Placing.—*

403—Pumping Concrete.—In giving some definite suggestions for pipe layout in the pumping method of concrete placement, all of which are sound and necessary, the most important item has been omitted. The pipe line leaving the pump must be straight and substantially horizontal for a distance equal in volume to several ejections of concrete—20 ft has been found none too much. A sudden change in direction near the pump has been found to cause a building up of resistances sufficient to stop the pump. With proper layout of the pipe line, pumping was found economical and highly satisfactory in placing all the concrete in the tunnel part and considerable volumes in the open-cut part of the section between 40th and 47th streets of the Sixth Avenue Subway in New York City.

406—Pneumatically Applied Mortar.—(*e*) This is the only reference to volumetric mix proportioning. However, it may be advisable to retain this method for pneumatically applied mortar and other plasters, if both cement and sand are measured in similar manner. A cubic foot of "dry and loose" cement weighs less than 50 lb.

The prohibition against the use of square reinforcement bars is not warranted if deformed bars are used (406(*m*)), since the shapes of square and round deformed bars are practically the same.

413–S—Compacting.—The recent use of non-absorbent coatings for forms, as well as steel forms, to obtain smoother surfaces, increases the water gain of concrete during large placing operations, as well as greater numbers of water bubbles on the exposed surfaces in spite of vibration-placing methods.

415–S—Depositing in Cold Weather.—A preference for the more finely ground cements for winter concrete use might be stated. A simple test for the existence of frost in concrete is to apply a plumber's torch against a small area from which the form is removed. Sudden signs of water upon such heat application are a definite proof of frost, showing that the forms must not be removed. The serious danger of alternate freezing and thawing, and rules for securing complete and monolithic setting of the concrete, if permitted to freeze, should be included.

*Chapter V—Details of Design and Construction.—*

502—Splicing of Reinforcement.—The specified lap method of placing rods in contact and wiring is a departure from accepted practice. For years, the field engineers have argued with lathers and insisted on the minimum separation of rods even at splices. The intent was to transfer stress through the concrete by bond resistance. It now may be argued that if rods are to be

placed in contact, the computed bond value must be assumed over a part (say 75%) of the lapped rods, since intimate bond contact cannot be expected over the entire perimeters. Is it the intent of the specification to increase the present standard splice lengths?

508—Future Bonding.—A simple method for protecting protruding bars has been a covering of waterproof fabric with incasement in cinder concrete. When the bars are for column extensions, either a solid metal cap flashed into the roofing or a pitch pot must be provided to prevent water from entering the concrete roof slab.

513—Location of Joints.—Joints should be located at places where cracks have been found. In concrete walls for buildings, a blind joint, in the form of a monel metal or aluminum strip or tee, should be placed vertically under each edge of all windows, to the floor level. Cracks in spandrel beams, usually resulting from the torsion induced by slab deflection, can be controlled by properly reinforcing the faces of the beams with longitudinal rods. Cracks in the exterior faces of top-story, exterior columns can be controlled by special rods in that face hooked into the top of the roof slab. Such recommended details can well be brought to the designers' attention in these specifications.

508–S—Location of Joints.—Although it is evident that columns must be poured and permitted to set and shrink before pouring the floor system, the specified method, keeping in mind that normally the concrete in columns is wetter than average (unless the inspector has perfect control), results in an accumulation of laitance at the top of each column. In some observed cases this layer was a full inch thick. Even a layer of paper thickness is undesirable in a structure where columns are assumed to be part of a rigid frame. Perhaps it might be advisable to assume pin connection at the top of columns and design accordingly; or, design the column forms for a little larger pressure and pour monolithically with the floor system.

*Chapter VI—Waterproofing and Protective Treatments.*—

602—Watertightness of Concrete.—In tests reported by the writer,[56] the relative permeability of concrete with various admixtures was no better than that of concrete with an addition of cement equal in cost to the admixture. However, there has been so much stress lately on maximum strength that the necessity of density has been relegated to the background.

603—Presence of Cracks and Joints.—Proper distribution of reinforcement for crack control must include uniform distribution. Cracks will occur in concentrated form at the weakest section. Proper distribution of reinforcement does not provide any weakest section.

611—Waterproofing with Portland Cement Mortar.—Much difficulty is experienced with water percolation in steel-incased concrete structures along seepage channels formed by the shrinkage of the concrete from the steel surfaces. Attempts to stop such leakage in subway walls and roofs, especially where the structural steel is not fully incased, by grouting cement under pressure, have not proved successful, chiefly because the cement particles are too large. Recently, such leakage has been stopped by grouting with bentonite

[56] *Proceedings*, Am. Soc. C. E., Vol. XLVIII, January–December, 1922, p. 1811.

(colloidal diatomaceous earth), because the finer particles entered the seepage planes and expanded with moisture absorption to clog the channels. Surface-treatment waterproofing cannot be expected to stop leakage under pressure permanently.

613—Waterproofing with Surface Treatments.—Failure of bituminous waterproofing coatings on subsurface structures has been noted in locations where gasoline or other light oils have been permitted to escape into the ground. These oils act as ready solvents of the bituminous materials, and will leak through the concrete with the water. Similar difficulty has even been found where concrete walls were waterproofed with a layer of brick laid in mastic.

*Chapter VII—Surface Finishes.—*

704—General.—The subject of forms cannot be considered complete without the inclusion of the recently developed "absorptive" form lining by the U. S. Bureau of Reclamation. The resulting surface is free of air and water bubbles, perfectly dense, uniformly textured, and most pleasing. This type of form shows practically no fins and eliminates all the objections to the plywood and pressed wood materials.

706—Monolithic Ornament.—Monolithic ornament, to avoid high cost, must be in the shape of recesses in the concrete. Projections can be provided only by building up a double form, and then perfect matching between adjacent pours is difficult to obtain. A recent contract in New York City specified the special manufacture of horizontal, uniformly spaced fins, $\frac{1}{4}$ in. wide by $\frac{1}{2}$ in. high. In spite of an expensive form construction, the result was far from a success, except possibly to the artistic temperament of the specification writer.

*Chapter VIII—Design.—*

804—General.—(*c*) The use of reinforcement in compression under the assumption of straight-line stress variation from neutral axis to compression surface has not been economical. The new recommendation doubles the computed value (with a maximum allowed stress of 16,000 lb per sq in.), but should be further justified by more than the ties or anchors. It is not compatible with the requirements of statics under the design assumptions. Compression reinforcement is often provided in the form of short, straight rods; the sudden discontinuity in the compression stresses makes it doubtful whether one can rely upon the recommended value.

804.—(*e*) If the transverse reinforcement over girders is designed to carry the slab load as a cantilever, it should be made clear whether the normal slab reinforcement for the same area of slab may be omitted. There also should be a restriction to maximum rod spacing in the transverse layer to twice the slab thickness.

809—General.—(*c*) If concrete slabs built into masonry walls are to be considered as restrained, the recent development of requiring a 1-in. air gap between the edge of the slab and the masonry must be omitted. It has been found in wall-bearing housing construction that cracks will develop in the masonry, horizontally along each floor slab, apparently due to the deflection or shrinkage of the concrete slab. Provision of the air gap has eliminated such cracking, in spite of the fact that load tests on such concrete slabs under 150% of the designed live load show substantially no deflection.

818—Types of Web Reinforcement in Beams.—If the full shear is to be taken by stirrups for $v = 0.06 f_c'$, the beams will be so full of steel that there is little room for the concrete—certainly not enough for it to develop the bond of all the steel. For shallow beams (12-in. total depth is very common in the low-rental housing designs), the requirement of paragraph 819 (*c*) is scarcely practicable; stirrups are necessary every 4 in., and there is no necessity for any computation of spacing.

819—Design of Web Reinforcement in Beams.—For vertical stirrups, the writer has used a simpler formula[57] that permits reading the stirrup spacing, for most beam loadings, on the slide rule.

In Eq. 3, since $V' = v' \, b \, j \, d$, and $f_s$ is 16,000, $s$ equals $\frac{3{,}520}{(v - v_0) \, b}$ for $\frac{3}{8}$-in. round stirrups, and $\frac{6{,}400}{(v - v_0) \, b}$ for $\frac{1}{2}$-in. round stirrups. In the foregoing $v$ is the total unit shear and $v_0$ is the unit shear taken by the concrete. The factors $F$, in $s = \frac{F}{v - v_0}$, for various beam widths are shown in Table 13. For double

TABLE 13.—FACTOR F FOR VARIOUS BEAM WIDTHS

| Diameter of rod, in inches | Factor F, for the Following Values of *b*: | | | | | |
|---|---|---|---|---|---|---|
| | 8 in. | 10 in. | 12 in. | 16 in. | 20 in. | 24 in. |
| 3/8 round.......... | 440 | 352 | 293 | 220 | 176 | 147 |
| 1/2 round.......... | 800 | 640 | 533 | 400 | 320 | 267 |

stirrups, the factors are doubled. The spacing at the support is the quotient of the factor divided by the unit shear taken by the stirrups. Successive spacings are found by reducing the denominator by the decrease in total unit shear in the spacing distance.

823—General.—In recent designs of concrete housing, the writer has found the use of welded wire fabric sheets in place of individual rods advisable. There are few available data on the bond value of such reinforcement. The usual practice in road work is to lap sheets by one spacing, expecting full development. The use of this type of fabric is being studied for walls and footings. In the last item, of course, bond stress becomes an important consideration. In slabs, the closer spacing of smaller rods, the very high elastic modulus (70,000 lb per sq in. minimum), and the welded spacers have all been proved efficient. A double span of 15-ft clear spans—designed for 60-lb live load with a tensile stress of 27,000 lb per sq in., total slab thickness of 5 in., fabric placed with $\frac{3}{4}$-in. clearance, and concrete testing at 3,300 lb in 28 days—showed no measured deflection under a test load of 100 lb per sq ft on both spans.

840—Beams in Flat Slab Construction.—Many flat slab designs have been built successfully with the spandrel beams equal in depth to the sum of the slab and drop thicknesses. Such spandrel beams are not designed to carry any part of the slab load, depending entirely on the exterior column band for such

[57] *Engineering News-Record*, Vol. 101, 1928, pp. 778–779.

transfer of load to the columns. Where deeper spandrel beams are used, designed to carry a part of the slab load, cracks are found in the stems (definite signs of torsion strains).

841—Column Capitals—Brackets—Drops.—For light loads, flat slab designs have been made and successfully executed with no slab drops, and with the column capital replaced by either a steel plate (usually with a steel or pipe column) or a steel beam grid.

843—Arrangement of Reinforcement.—Corner reinforcement, similar to that recommended for two-way slabs (see 811(*c*)), should be required for flat slab design. Similar corner diagonal cracks have been noted in flat slab buildings.

852—Limiting Dimensions.—The minimum of 10 in. for main columns (in place of the present customary 12 in.) is necessary for the design of low-story residence buildings, where the unsupported column length is often less than 7.5 ft. However, the spiral reinforced column should not be permitted for columns less than 16 in., round or square.

There seemed to be only two additions needed to make the Report complete, and they are: (1) Limitation of maximum spacing of rods in slabs; and (2) percentage of transverse or temperature rods in slabs (although the requirement for retaining walls is given in 877).

The writer would recommend: (1) A maximum spacing of twice the total slab thickness but not more than 6 in. for slab rods; and (2) temperature rods to be spaced 18 in. apart in floors and 12 in. in roofs, wired to the main reinforcing and only placed in the sections where the main reinforcing is in tension, using rods equal in tensile strength to a minimum of 1,800 lb per rod (or 100 lb per in. in floor slabs and 150 lb per in. in roofs). Where welded wire fabric is used, the cross wires can be used for temperature reinforcement, if of sufficient area to meet the strength requirement and if lapped by one spacing of the main wires at the ends of each sheet width.

This discussion covers only a few of the many recommendations in the Report, and only those with which the writer is not in full agreement as to statement or definiteness. In general, too high praise and thanks cannot be given to the members of the Committee for their excellent Report.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

# EARTHQUAKE STRESSES IN THE SAN FRANCISCO-OAKLAND BAY BRIDGE

## Discussion

BY FRANKLIN P. ULRICH, M. AM. SOC. C. E.

FRANKLIN P. ULRICH,[7] M. AM. SOC. C. E. (by letter).[7a]—Although past history shows that earthquake damage has been chiefly confined to structures of poor construction or poor design, there is no assurance that what engineers now consider good construction is immune from future earthquake damage.

There are several factors to consider in possible earthquake damage, including intensity of earthquake, nearness to center of disturbance, and condition of the structure, especially foundation conditions.

In an attempt to learn more about some of these factors, 62 strong-motion seismographs have been set up in Western United States. These instruments will operate during the strongest earthquakes, giving information vital to structural design, such as acceleration, amplitude of motion, period of oscillation and duration of shock and its various parts. Although these instruments have been in operation only a few years, some surprising information has been obtained. In the El Centro (Calif.) shock in 1940 these instruments recorded a resultant horizontal acceleration of 38% of gravity. Some schools in this district that were designed to withstand continuous horizontal accelerations of 10%, and other buildings where the earthquake factor was not especially considered in the design, came through the shock with no serious structural damage. There is a factor in every structure which might be called the "inherent strength," and which is capable of resisting earthquake stress, effectively, to some unknown degree. Hence, a structure that is designed to resist an earthquake acceleration of 10% may, and no doubt would, withstand an earthquake acceleration several times that amount, providing it accompanied relatively short-period motion, as it usually does.

The second factor, mentioned previously, is the nearness to the epicenter. It is quite certain that all normal structures set directly on an earthquake

NOTE.—This paper by Norman C. Raab, M. Am. Soc. C. E., and Howard C. Wood, Assoc. M. Am. Soc. C. E., was published in October, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: December, 1940, by Maurice A. Biot, Esq.

[7] Chf., Seismological Field Survey, U. S. Coast and Geodetic Survey, San Francisco, Calif.

[7a] Received by the Secretary February 7, 1941.

fault, such as the San Andreas, would be seriously damaged by an earthquake of the intensity of that in 1906. It would seem equally certain that a relative lateral motion of two adjacent piers equal to that measured in the 1906 earthquake would seriously damage the San Francisco-Oakland Bay Bridge.

The Fault Map of the Seismological Society of America does show a "probable active fault" along the west side of Yerba Buena Island, and in recent years there has been at least one minor shock along this fault, near the Oakland Mole. However, in this case major active faults are situated relatively close on either end of the bridge, and it would appear to be reasonable logic to expect that large earth movements would occur along those faults and not on a minor fault between these major faults. Minor earthquakes are frequent in this region, but are generally confined to the major fault zones.

Information on the seismicity of this region and other regions is being obtained for the engineer and designer through the network of sensitive seismograph stations and through the extensive program of reports of felt earthquake shocks.

The third factor of foundation conditions and condition of structure is perhaps not as vital in this bridge as on normal buildings where uneven settlement would impose excessive stresses in some of the structural members, or structural deterioration would weaken the structure to a danger point—especially at the time of a strong earthquake. In this bridge the foundations of the west piers extend to rock and the foundations of the east piers extend to rock or comparatively solid strata so that there should be no settlement problem. The durability of concrete under salt water for a long period of time might be an uncertain factor, but tests made on the concrete used in these bridge piers showed a very satisfactory resistance to this deterioration.

*Summary.*—(1) In the light of past experience and the present knowledge of destructive ground motion the assumed earthquake factor of 10% at the period of 1.5 sec would probably be sufficient to resist an earthquake as great as would be expected in this region; (2) with major active faults at some distance from each end of the bridge it would be logical to believe that the bridge will not be subjected to uneven movements of adjoining parts of the structure; (3) with the piers extending to bedrock, or solid strata, there should be no settling of the bridge, especially uneven settlement; and (4) the weakening of the structure through deterioration of the concrete is an uncertain factor, but has been carefully checked in preconstruction tests.

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DISCUSSIONS

# AN INVESTIGATION OF STEEL RIGID FRAMES

## Discussion

By Messrs. LaMotte Grover, and William R. Osgood

LaMotte Grover,[14] M. Am. Soc. C. E. (by letter).[14a]—The tests described have made an excellent start in the investigation of stresses in rigid-frame knees. The authors are to be commended for their thorough interpretation of results and especially for giving structural designers practical rational methods of stress analysis that will yield sufficiently accurate results at least for knees of shapes and proportions similar to those tested, when they are subjected to static loading. These methods have the virtue of focusing the attention of the designer upon the structural behavior of such knees and the stress distribtion in them, thus providing at least some orientation in the matter of designing for dynamic loadings.

It is important to recognize that this investigation, together with its companion investigation at the National Bureau of Standards, covers too small a range of knee shapes to warrant conclusions as to the best general type, such as quite a number of structural engineers seem to have assumed after reading the reports; although no such conclusions are represented by the investigators. None of the tests has included a welded curved knee. The welded knee tested by the Bureau of Standards had straight sloping inner flanges.

It was perhaps inopportune that the Lehigh riveted curved knee was proportioned similar to the one tested by the Bureau, which had confirmed the belief that serious elastic instability is inherent in such widely expanded and flexible knees as have been used for some building frames, usually to conform to architectural plans. It might have been better to strive for proportions that would provide the best distribution of stresses possible with a curved inner flange of smaller radius, decreasing the large flexible web area and increasing the compression flange stiffness. In the design tested, flange angles were used alone without cover plates.

Note.—This paper by Inge Lyse, M. Am. Soc. C. E., and W. E. Black, Jun. Am. Soc. C. E., was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1941, by C. J. Posey, Assoc. M. Am. Soc. C. E.; and February, 1941, by W. J. Eney, Assoc. M. Am. Soc. C. E.

[14] Structural Welding Engr., Applied Eng. Dept., Air Reduction Sales Co., New York, N. Y.

[14a] Received by the Secretary January 30, 1941.

In comparing the general behavior of the square-knee frame with that of the curved-knee frame, it must be borne in mind that the members themselves in the square-knee frame were considerably deeper and stiffer.

The riveted square knee, as tested, was designed principally with the viewpoint of facilitating fabrication, which is of paramount importance. High concentrations of stress were found at the sharp inside corner but it was concluded that these were due largely to extreme concentration of bearing at that point, resulting from shims driven in between the face of the vertical leg and the outstanding legs of the compression flange angles of the girder, where it is very difficult to obtain reliable bearing in riveted construction. Little significance was attributed by the authors to the general effect of the sharp reentrant angle. This conclusion seems highly questionable in view of the drastic effects upon stress distribution that have been observed for such sharp angles or "notch effects" in other investigations. Even higher local concentrations of stress might have been found if the corner areas had been explored more fully by differential tensometer readings.

It is questionable whether the normal stresses on the sections perpendicular to the axes of the leg and the girder (Fig. 9) carry much significance within the square knee and immediately adjacent to it. Normal stresses on diagonal or radial sections passing through the inner corner would seem more significant in view of the orientation of the maximum principal stresses in this vicinity, a fact recognized by the authors in discussing Fig. 14($a$).

One of the conclusions was that any effect of plastic flow or permanent set and slippage of rivets upon the behavior of the frame as a whole was negligible. However, Table 3 shows an appreciable decrease in observed horizontal reaction and an approximately corresponding increase in observed center moments and deflections as compared with computed values, for both types of frames. Although Fig. 12 indicates elastic behavior throughout the range of loading, it must be remembered that the loading was repeated many times in gathering all the data for the investigation (because of the limited number of only six tensometers available for use). Initial loadings, prior to taking the readings for Fig. 12, could have produced permanent relative rotations between the axes of the intersecting members at the knees, which would result in initial no-load stresses for the next following tests. Under these circumstances, elastic behavior would be expected unless the initial load were exceeded. The effect of rivet slippage, and permanent set or plastic flow, might be more pronounced in a multiple-span frame with three-branch knees at the intermediate supports.

As inferred at the beginning of this discussion, the tests conducted thus far cannot be considered much more than a good start in an adequate investigation of rigid-frame knee design. To gain an impression of the ground that remains to be studied, as well as an indication of the nature of further investigations that are needed, it should be helpful to review briefly what has been done by other investigators in this field, and to point out wherein their conclusions seem to be reinforced by the results of the Lehigh tests and also where there appears to be disagreement.

In Europe, development as well as research in rigid-frame construction has been based for the most part upon welded details because of their inherently greater effectiveness in providing rigid connections with a minimum amount of material.

In Belgium and Germany extensive studies of stress distribution in rigid-frame joints of various types have been made along with many tests under static and fatigue loadings, of large and small scale steel models of built-up flanged specimens as well as plane models. Meticulous determinations of stresses and their distribution have been made throughout the webs and flanges, and transversely across the latter, by means of tensometer readings on short gage lengths—some of them differential readings, where stresses vary rapidly. These quantitative readings have been supplemented by photoelastic studies and also by the determination of stress patterns as revealed by the cracking under tensile stress of a brittle, tightly adhering coating of varnish that is applied to specimens before loading, for investigation of tensile stress, or applied while the specimen is under load, for investigation of compression stress. Some of these investigations have included three-branch joints simulating the rigid connection of a flexural member to a column or chord carrying considerable direct stress.

Investigators agree that in knees of all types, as indicated by mathematical analyses, the neutral axis lies between the centroidal axis and the inner flange, and the distribution of normal stresses departs from a straight-line distribution within the knee. Both tangential and radial stresses are concentrated at and near the inner flange, increasing considerably as the curvature or haunch angles of this flange become sharper.

There is general agreement that under usual conditions of design loading the critical section for tensile stresses as well as compression lies between the diagonal section through the center of the knee and the section where the knee joins one of the members, usually close to the latter if not directly at it. This suggests the possibility of fatigue failure at these critical sections under repeated stress, even though the frame would fail by buckling of the compression flange well within the knee under a static loading of considerably greater magnitude.

Quite a number of investigators have concluded that abrupt angles or sharply curved inner flanges must be avoided to eliminate high, local stress concentrations which are especially objectionable under dynamic loading. As in the Lehigh investigation, they have found that the radial component of the stress in a curved compression flange, which becomes greater as the curvature becomes sharper, causes transverse bending or cupping of that flange with consequent concentration of flange stresses near the junction with the web, and reduction of the effective flange area. Because of unavoidable lack of perfect symmetry of the cross section, due to slight inaccuracies in fabrication, an eccentricity develops which accentuates the effect of the radial compression force. However, when quite large radius haunches are used, the knee area is so large and the web so flexible, even though stiffened, that it will buckle under comparatively small loads and permit lateral deflection and buckling of the compression flange. It is necessary, therefore, to balance a design between

consideration of stress distribution on the one hand and elastic stability on the other.

Conclusions reached from the Lehigh curved-knee test attribute major importance to the rapid change of section occurring at the junctions of the members with the knee. Belgian and German investigators have concentrated much of their attention upon this same feature. Their objective has been to reduce the rate of change in curvature at the beginning of curvature, and to avoid discontinuities at these critical sections, thereby eliminating stress concentrations or rapidly increasing principal stresses, which are distinctly revealed in the stress-line patterns by sharply curved lines. Stress lines, especially those along the inside edges of the knees, have been studied in many models involving rectilinear inner edges or flanges and geometric curves of various shapes and sizes, parabolic, hyperbolic, elliptical and sinusoidal (symmetrical and unsymmetrical or deformed, depending upon the relation between the direct stresses and bending stresses in the members joined).

An assumption commonly made by proponents of knees with rectilinear borders is that bending stresses are largely channeled in the flanges, and that the logical arrangement is that of straight flange elements combined with stiffeners in such a way as to form triangular force systems in equilibrium somewhat similar to those in trusses. Some of those technicians who favor curved inner flanges oppose the use of stiffeners or other special appendages for the transmission of stresses, because they have been held partly responsible for rather sharp transverse deviations in stress paths in the flanges to which they have been joined. It is apparent from test results, however, that curved compression flanges and adjacent web areas must be stiffened to reduce transverse bending of the flanges and to prevent lateral flange deflection and buckling in the web, unless it is practical to make the proportions of the width, thickness, and curvature of the flange and the thickness of the adjacent web such that elastic stability and good transverse stress distribution are assured. L. C. Maugh, Assoc. M. Am. Soc. C. E., has concluded from model tests at the University of Michigan, Ann Arbor, Mich., that stiffeners are required when the quantity $\frac{b^2}{4\,t\,r}$ exceeds 0.3 ($b$ and $t$ being the width and thickness of flange and $r$ its mean radius of curvature). This expression does not evaluate the effect of web thickness. Professor Maugh feels that stiffeners should extend the full depth of the web so that the compression flange will receive lateral support from the tension flange through the stiffened web. Other investigators consider it sufficient to provide only small triangular transverse corner diaphragms that stiffen the compression flanges and adjacent web areas. A combination of the two types of stiffeners seems expedient, with all of them welded to the web and to the compression flange.

Unless a very decided advantage can be shown for the curved knee in the way of eliminating high stress concentrations that would contribute to real structural damage under repeated stress at the critical sections, and unless that advantage would be reflected in a substantial saving in weight of material, the economic preference will undoubtedly be in favor of the square or recti-

linear knee for riveted rigid-frame bridges as well as buildings. Any such economic preference for the rectilinear knee is much less pronounced in the case of welded fabrication. One must also consider whether there is a general architectural preference for either the curved knee or the sharp-corner knee, one over the other.

The several rational methods of design that have been formulated by various investigators provide a much better insight into the behavior of rigid-frame knees than that afforded by conventional methods applied to many other commonly used structural details. Stresses in the members at sections only a short distance away from the knee have been found to conform well to computations by conventional theory. Designers should not hesitate, therefore, to use rigid frames.

Nevertheless, further tests of a comprehensive nature are needed in the United States to confirm or refute the conclusions of foreign investigations and perhaps to correct misinterpretations that have been made of meager tests in this country. This is the more important now that the completion of European tests has been interrupted by war activities. More information is needed about the influence of knees upon the behavior of a frame as a whole, especially the effects of plastic deformation or permanent set in cases of overloading, small settlements and horizontal movements of supports; also information to enable a better evaluation of the significance and importance of comparatively high shearing stresses that are known to exist within rigid-frame knees.

It is not merely the matter of a possibility of saving a comparatively small amount of material in the knee itself, as opposed to a generous proportioning of that element. The shape and type of knee and its rigidity exert a marked influence upon the stresses at the critical sections, adjacent to the members of the frame, which members, therefore, are likewise influenced at least locally. It is very desirable to evaluate these influences under repeated stress.

It may be that foreign investigators, as well as some students of stress analysis in the United States, have very greatly exaggerated the importance of sharp deviations in stress paths in the knees of rigid-frame bridges. Practical minded American engineers would surely not be willing to add much cost to their structures merely for the sake of an elegant solution to a stress problem. However, if there is really no practical value in refinements in the proportioning of rigid-frame knees, this should be proved definitely once and for all.

In many other structural details it has been proved quite important to avoid forms of details that cause abrupt disturbances in stress paths at critical points, even in statically loaded structures. It is evident from test results that there is a distinct correlation between fatigue strength and stress distribution. Therefore, it is logical to pursue further studies of rigid frames largely through fatigue tests. The most favorable proportions for each type should be determined for comparison to evaluate relative advantages. This can be done more quickly and at less expense if the fatigue tests are supplemented by static tests of models coated with brittle varnish to reveal the stress patterns and thereby guide the investigators more rapidly to a determination of the best details for resisting fatigue stresses, as well as static load stresses.

The program of further tests should be arranged objectively and without prejudice. In too much of the research work that has been done already, one type or another has been virtually disqualified at the start, as a result of subjective or premature conclusions.

WILLIAM R. OSGOOD,[15] M. AM. SOC. C. E. (by letter).[15a]—It is very much worth while to have tests checked in different laboratories. Such checking or duplication is common in many scientific laboratories such as laboratories of physics, chemistry, medicine, etc. It is not so common in engineering laboratories, possibly because of the frequently greater costs involved. When it can be done, however, it is to be welcomed. The authors have confirmed test results obtained at the National Bureau of Standards and have introduced ideas of their own. The paper contains much that can be a source of only satisfaction to those interested in rigid frames. Some points will bear discussion and comment.

The authors state (see "Synopsis") "In the square knee, a concentration of stress existed at the inner corner but was found to be of minor importance." In view of the fact that a comparable square-knee frame tested at the National Bureau of Standards[3] failed by buckling of the outstanding flanges of the angles at the inner corner, it would be interesting to know on what grounds the authors base their statement.

The results shown in Fig. 20 confirm the validity of the theory developed at the Bureau of Standards[4,8] even better than the tests which led to that development, at least so far as the stresses in the flanges are concerned. Before making too much of disagreement between theory and test, it is suggested that equilibrium of the part of the frame on one side of a circular section be considered. The theoretical stresses on such a section are in equilibrium with the applied loads on either side of it. Are the observed stresses in equilibrium with these loads? If not, the observed stresses are manifestly in error; and if the necessary adjustment of the observed stresses to bring about equilibrium is of the order of the difference between observed stress and theoretical stress at any point, then it may be impossible to draw any conclusions as to how imperfect the theory is.

The "Observed Position of Neutral Axis" in Fig. 20 may be misleading. If this axis is the axis of zero normal stress on radial sections, it bears no relation to the points of zero normal stress on the circular sections shown in the figure.

The authors make the statement that for a curved knee "critical sections for the direct stress at the knee occur within 15° from the points of tangency of the curved flange." The implication is that these sections will always occur

[15] Materials Engr., National Bureau of Standards, Washington, D. C.

[15a] Received by the Secretary February 5, 1941.

[3] "Strength of a Riveted Steel Rigid Frame Having Straight Flanges," by Ambrose H. Stang, Martin Greenspan, and William R. Osgood, M. Am. Soc. C. E., *Research Paper No. 1130, Journal of Research*, National Bureau of Standards, Vol. 21, 1938, p. 269.

[4] "Strength of a Riveted Steel Rigid Frame Having a Curved Inner Flange," by Ambrose H. Stang, Martin Greenspan, and William R. Osgood, *Research Paper No. 1161, loc. cit.*, p. 853.

[8] "A Theory of Flexure for Beams with Nonparallel Extreme Fibers," by W. R. Osgood, *Journal of Applied Mechanics*, September, 1939.

there, regardless of the positions of the applied load. The writer does not think that this will be the case. If the approximate rational method of determining the critical sections is used, as suggested in reference 4, the critical curved-flange sections for the loading shown in Fig. 11 occur at 9°.8 from the upper point of tangency and 7°.4 from the lower point of tangency. These sections are close to the sections of maximum stress shown in Fig. 18. If the ratio of $P$ to $H$ in Fig. 1($b$) had been considerably larger, for example, it is probable that the upper critical section would have been found at an angle greater than 15° from the point of tangency of the curved flange.

The theory presented by the writer [8] in 1939 is not of the simplest, as the authors intimate, but on the other hand, it is not at all difficult to use. Three sets of curves are given[8] in which the ratio of stress computed by the ordinary beam theory to stress computed by the tapered beam theory is plotted against the ratio of flange area to total circular sectional area. It is only necessary then to compute the stress by the ordinary beam theory, enter the chart at the proper ratio of flange area to total area, and read the ratio of the two stresses at the curve drawn for the proper angle measured from the point of tangency of the curved flange.

It is perhaps significant that the authors do not even mention radial stiffeners and buckling of the web of the frame having the curved inner flange. Evidently there was no evidence of buckling due to radial compressive stresses, even though the average compressive fiber stress in the curved flange was as high as 24,000 lb per sq in. and the highest nearly 30,000 lb per sq in. (Figs. 18 and 10). This finding is of importance because it has not infrequently been the practice to stiffen the web radially, and "in massive doses." The authors' test was not carried to destruction, it is true; but the Bureau of Standards' test on a similar frame was, and there was no evidence that the web buckled. The radial compressive stresses were low, with probable maximum values not exceeding 12,000 or 13,000 lb per sq in. at spots. "Failure occurred suddenly, by lateral deflection of the curved inner portion of the frame," [4] that is, as a sidewise displacement of the inner flange as a whole, not as a curling over of the flange. This kind of failure can only be prevented by lateral bracing—radial stiffeners are useless to prevent it.

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DISCUSSIONS

# RELIABILITY OF STATION-YEAR RAINFALL-FREQUENCY DETERMINATIONS

## Discussion

BY MESSRS. MERRILL BERNARD, AND CHARLES F. RUFF

MERRILL BERNARD,[29] M. AM. SOC. C. E. (by letter).[29a]—Efforts to inject mathematical precision into the treatment of hydraulic problems that rely upon precipitation for solution are timely. Too often progress in fields of applied science depends upon the ability to bolster an inadequate sampling medium with assumptions that are never wholly satisfying. Such was the necessity faced by the Miami Conservancy District in 1916. Until that time rainfall records were considered to be so many independent point experiences, and only an occasional attempt had been made to organize the data on the basis of the storm as the unit.

In the design of structures, the engineers of the Miami project found it necessary to evaluate the damage to be expected from the highly infrequent storm and flood that might conceivably damage or destroy the protection works. To do this intelligently it was necessary to know the frequency with which rain fell at various intensities and durations. At the time there were only a few rainfall stations in Eastern United States with records greater than 30 years. Also, the number of stations in the network was admittedly inadequate. Expediency demanded the means of extrapolating the frequency of the greater values to a period longer than the actual record and in reasonable accord with the estimated physical life of the protection system. Thus, the "station-year" method of adding the records at a number of stations to obtain an equivalent single station record was developed. It is believed that an understanding of this paper will be enhanced by the following attempt to present a simplified concept of the station-year method for determining rainfall frequency.

There is little theoretical justification for combining an indiscriminate number of concurrent rainfall records to arrive at the record for a single station, equivalent in length to the sum of the station years; and yet, where the

NOTE.—This paper by Katharine Clarke-Hafstad was published in November, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1941, by Paul V. Hodges, M. Am. Soc. C. E.; and February, 1941, by Messrs. C. S. Jarvis, and Howard W. Brod.

[29] Superv. Hydrologist and Service Coordinator, U. S. Weather Bureau, Washington, D. C.

[29a] Received by the Secretary February 13, 1941.

occasional opportunity arises to allow such a comparison, the synthesized record often compares favorably with the actual. The writer will resort to a reasonably rational but wholly artificial means of attempting to arrive at a better understanding of the "composite record" or "station-year" method, its physical and statistical limitations, and any success it may have had in guiding engineering judgment.

Visualize a large table having an area confined within a raised edge measuring 12 ft by 12 ft and containing 20,736 sq in. The table top is intended to represent a region of meteorological homogeneity. By definition, such a region is one, all parts of which can be said to have the same rainfall regimen.

In reality there are no areas of appreciable extent that conform to this specification. Homogeneity implies, first, a complete stability in the relationships between the meteorological factors involved in the mechanism of a storm and, second, a consistent action from storm to storm of surface influences, such as mountain barriers, abrupt changes in elevation, and the proximity of appreciable water surfaces. It is understood, then, that the suggested demonstration

TABLE 6.—ANALYSIS OF STORM VISITATION

| Ring diameter, in in. | Area, in sq in. | Incremental area, in sq in. | Average depth, in in. | No. of times contained in total area | Average No. of throws to make one hit | Minutes at end of which one hit is made | Average No. of hits at the time first hit in smallest circle is made | No. of hits at end of 17,800 min | $\Delta A$ | $\Delta B$ | $\Delta C$ | Summation of Cols. 10, 11, and 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) | (11) | (12) | (13) |
| (*a*) STORM *A* (AVERAGE SEQUENCE OF THROWS, ONE THROW EACH 12-MIN PERIOD) | | | | | | | | | | | | |
| 3 | 7 | 7 | 13.0 | 2,960 | 1,480 | 17,800 | 1 | 1 | 1 | ... | ... | 1 |
| 5 | 20 | 13 | 9.3 | 1,600 | 800 | 9,600 | 2 | 2 | 2 | ... | ... | 2 |
| 8 | 50 | 30 | 7.5 | 690 | 345 | 4,150 | 4 | 4 | 4 | 6 | ... | 10 |
| 12 | 113 | 63 | 4.0 | 329 | 164 | 1,970 | 9 | 9 | 9 | 29 | 41 | 79 |
| 16 | 201 | 88 | 1.0 | 236 | 118 | 1,420 | 13 | 13 | 13 | 70 | 341 | 424 |
| (*b*) STORM *B* (AVERAGE SEQUENCE OF THROWS, ONE THROW EACH 4-MIN PERIOD) | | | | | | | | | | | | |
| 3 | 7 | 7 | 8.3 | 2,960 | 1,480 | 5,920 | 1 | 3 | 3 | 3 | ... | 6 |
| 5 | 20 | 13 | 7.3 | 1,600 | 800 | 3,200 | 2 | 6 | 4 | 6 | ... | 10 |
| 8 | 50 | 30 | 6.3 | 690 | 345 | 1,380 | 4 | 12 | 5 | 12 | ... | 17 |
| 12 | 113 | 63 | 4.3 | 329 | 164 | 656 | 9 | 27 | 8 | 27 | 35 | 70 |
| 18 | 254 | 141 | 1.7 | 147 | 74 | 296 | 20 | 60 | 12 | 60 | 215 | 287 |
| 24 | 452 | 198 | 0.5 | 105 | 52 | 208 | 28 | 84 | 14 | 84 | 539 | 637 |
| (*c*) STORM *C* (AVERAGE SEQUENCE OF THROWS, ONE THROW EACH 1½-MIN PERIOD) | | | | | | | | | | | | |
| 3 | 7 | 7 | 5.7 | 2,960 | 1,480 | 2,220 | 1 | 8 | 6 | 15 | 8 | 29 |
| 5 | 20 | 13 | 4.9 | 1,600 | 800 | 1,200 | 2 | 15 | 8 | 21 | 15 | 44 |
| 8 | 50 | 30 | 4.3 | 690 | 345 | 513 | 4 | 35 | ... | ... | ... | ... |
| 12 | 113 | 63 | 3.3 | 329 | 164 | 246 | 9 | 72 | 10 | 37 | 72 | 119 |
| 18 | 254 | 141 | 2.2 | 147 | 74 | 111 | 20 | 160 | 12 | 52 | 160 | 224 |
| 26 | 530 | 276 | 1.1 | 75 | 38 | 57 | 39 | 312 | 13 | 68 | 312 | 393 |
| 36 | 1,017 | 477 | 0.5 | 43 | 22 | 33 | 67 | 539 | ... | ... | ... | ... |

"region" is homogeneous only within the tolerances imposed by the foregoing considerations.

Next, prepare three sets of concentric rings of light metal, symmetrically spaced and held rigidly in place according to the specification in Col. 1, Table 6.

For the moment, ignore the artificiality of the medium, and visualize, in the sets of concentric rings, the isohyetal pattern of rainfall idealized as to shape. Usually such a pattern is elongated and irregularly elliptical. It is believed little would be added to the experiment if differences in pattern shape were included as a variable.

There are innumerable shades of difference in the characteristics of storms, and only a generalized classification is possible. A principal distinction lies in the inverse relation between intensity and the duration and areal extent of the storm. Three generalized storm types are pictured in the depth-area curves shown in Fig. 4. Storm *A* is the highly intense downpour of limited

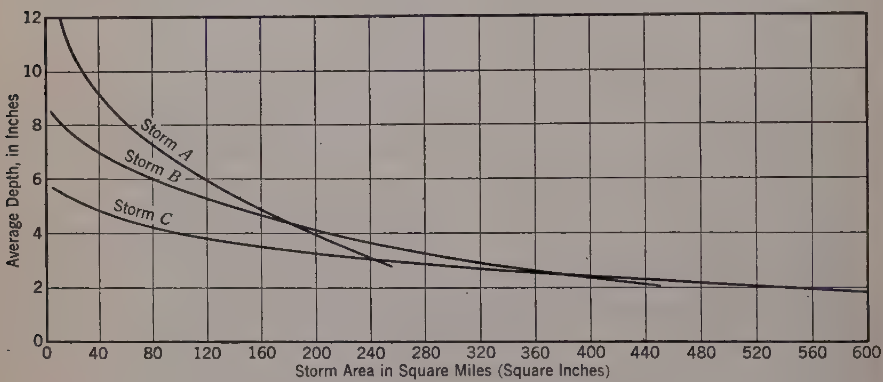


Fig. 4.—Area-Depth Curves, Hypothetical Storms

extent that produces the phenomenal depths of rainfall recorded in certain parts of the United States. Storm *C* is the typical "warm front" storm producing moderate intensities over considerable area; and storm *B* is of intermediate type, often combining the characteristics of types *A* and *B*.

In Table 6, consider the differences in the diameters to fix the widths of the concentric bands between the rings (isohyets) of each "quoit" (storm). It is realized that the duration of storms *A*, *B*, and *C* will vary considerably. It will not detract too greatly from the reality of the experiment if an average duration is adopted for all.

Although no attempt is made to simulate quantitative relationships found in actual storms, it will be reasonable to consider square inches in the model as equal to square miles of the prototype. Depths have been computed from Fig. 4 that are to be taken as averages for the concentric bands between the isohyets.

A region of meteorological homogeneity also can be defined as one in which a storm, having entered it, can center over any point within its limits with equal probability. Without going into mechanical detail, assume the means of injecting the storm quoits to, and on, the table with the impersonal result of mechanically thrown dice, or under the assumption that the quoit, having had energy applied to it, is under perfectly balanced compensative influences so that it can come to rest in any position on the table with complete fortuity.

Attention is now directed to Table 6, in which Cols. 1 and 2 are isohyetal diameter and enclosed area, respectively. Col. 3 is the area of the concentric band between a given ring and the next smaller one. Col. 4 is the assigned average depth of the concentric bands between isohyets. Col. 5 is the number of times the respective incremental areas are contained in the entire table area. (The circles could be adjusted actually to space by changing the shape or overlap, but this is not considered necessary to the purpose of the discussion.)

The following sequence of average storm visitation to the table (region) is adopted:

| Minutes | Storm | Minutes | Storm |
|---|---|---|---|
| 1 | *C* | 9 | *B* |
| 2 | *C* | 10 | *C* |
| 3 | *B* | 11 | *C* |
| 4 | *C* | 12 | *A* |
| 5 | *C* | 1 | *C* |
| 6 | *B* | 2 | *C* |
| 7 | *C* | 3 | *B* |
| 8 | *C* | etc. | etc. |

At the beginning of the initial minute (year) of the record, a small upright peg (station) is placed in an arbitrary position on the table (region). The experiment now begins under the foregoing schedule by putting the mechanical quoit thrower into action. The peg can be taken to mark the center of one of the 2,960 areas measuring 3 in. in diameter contained in the entire table top (Col. 4). It is possible that the first quoit thrown could center over it, or the entire 2,960 opportunities could be exhausted before the peg (station) was encircled by the 3-in. ring. If an indeterminate number of runs are made, it will be found that the peg will be ringed as often in the first half as in the last half of the series so that the most probable number of throws necessary to ring the peg would be the average, or 1,480.

With only one peg on the table, by far the greater number of throws will miss the peg entirely (no rain recorded at the station). The number of times the peg falls between the isohyets of greater diameter will be in proportion to the opportunity afforded by the greater area of the concentric bands between the rings (isohyets).

Col. 6, Table 6, gives the average number of throws under the schedule to bring the peg within the areas fixed by the various isohyetal diameters. Col. 7 gives, for each storm type, the average number of minutes at the end of which one hit was made in each of the concentric bands. Col. 8 gives the number of hits for each isohyet diameter within the average number of minutes taken to ring the peg in the smallest isohyet of each storm type. Col. 9 gives the number of hits within the average number of minutes taken to ring the peg with the smallest isohyet of the least frequent storm *A* (17,800 min).

It is now seen that the peg (station) has experienced through the record period of 17,800 min (months, years) the visitation of three types of storms in varying sequence, producing varying depths of rainfall. Although storm *A*

contributed the two greatest depths, the remainder of the depth-range was represented generally by all three storm types. In order, then, to organize the record at the peg (station), it is necessary to assemble the number of occurrences for each average depth from each type of storm, as shown in Cols. 10, 11, 12, and 13. These values are arranged in order of depth-magnitude in Col. 1, Table 7. The frequency with which one peg (station) will experience various depths of rainfall is given in Col. 3, Table 7.

TABLE 7.—ANALYSIS OF STORM FREQUENCY

| Average depths, in in. (1) | No. of hits (2) | Frequencies, F: One station (3) | Two stations (4) | Twenty stations (5) | Thirty-six stations (6) | Average depths, in in. (1) | No. of hits (2) | Frequencies, F: One station (3) | Two stations (4) | Twenty stations (5) | Thirty-six stations (6) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 13.0 | 1 | 17,800 | 8,900 | 890 | 494 | 4.3 | 70 | 254 | 127 | 13 | 7 |
| 9.3 | 2 | 8,900 | 4,450 | 445 | 247 | 4.0 | 79 | 225 | 112 | 11 | 6 |
| 8.3 | 6 | 2,960 | 1,480 | 148 | 82 | 3.3 | 119 | 150 | 75 | 8 | 4 |
| 7.5 | 10 | 1,780 | 890 | 89 | 49 | 2.2 | 224 | 79 | 40 | 4 | 2 |
| 7.3 | 10 | 1,780 | 890 | 89 | 49 | 1.7 | 287 | 62 | 31 | 3 | 2 |
| 6.3 | 17 | 1,050 | 525 | 52 | 29 | 1.1 | 393 | 45 | 22 | 2 | 1 |
| 5.7 | 29 | 614 | 307 | 31 | 17 | 1.0 | 424 | 42 | 21 | 2 | 1 |
| 4.9 | 44 | 404 | 202 | 20 | 11 | 0.5 | 637 | 28 | 14 | 1 | 1 |

The demonstration is thus complete for a record cycle of 17,800 min at the peg (station). If the cycle is representative, and if 13 in. is considered to be the maximum depth for the region, even if it were experienced in the first minute (month, year) of record, the peg (station) experience must continue throughout the course of the cycle if it is to be confirmed as a maximum value.

The analogy between the table top and the region is strengthened somewhat if the former is considered covered with green baize and the outline of each quoit (storm pattern) is made with chalk as each throw comes to rest. The result at the end of a complete series of throws will be a completely covered table top (regional map) with superimposed storm patterns giving the same impression as that gained by superimposing maps prepared by the Miami Conservancy District.[30]

If station 1 is 100 ft from station 2, it will be impossible for station 1 to have, say, 3 in. of rainfall without station 2 experiencing a fall somewhat near this quantity. In other words, the record accumulating at station 2 is not independent of that at station 1. As station 2 is moved away from station 1, its dependency upon the latter will become less and less until the two records no longer represent that of concurrent rainfall, although statistically both may have the same frequency curves.

Dependency may be illustrated on the table top and with the hypothetical storms $A$, $B$, and $C$. With station 1 at the center of storm $A$, the dependency of the record at station 2 for a storm of this type will reduce in proportion to the lengthening radius of storm $A$ to its outer limit of 8 in. (miles). With the peg (station) at the outer limit of the storm, a distance between stations 1 and

[30] "Storm Rainfall of Eastern United States," Miami Conservancy District, Eng. Staff, *Technical Reports*, Pt. 5, Revised Edition, 1936, Dayton, Ohio, Figs. 54 through 65.

2 must be equal to at least the diameter (16 in.) to insure complete independence of record. Under these requirements the minimum distance for storm *B* would be 24 in., and for storm *C* 36 in.

With this fact in mind locate peg (station) 2 at some arbitrary point on the table and at a greater distance than 36 in. from peg (station) 1. Now subject the table (region) to a series of storm visitations according to the established schedule. Because of the wholly fortuitous delivery of the quoits (storms), one station or the other will ring (record) the 3-in. isohyet of storm *A* for an average series of throws in a time period, equal, on the average, to one half of that taken to make the same hit with only peg (station) 1 on the table. If other pegs (stations) are added, the average time taken to ring the 3-in. isohyet of storm *A* in any series of throws will be equal to the result of dividing the period for one peg (station) by the number of pegs (stations).

The dependency relationship between stations is affected also by the relative importance of depth. Considering the diameter of the isohyets assigned to the various depths to be the minimum distance between stations, it is seen from Fig. 5 that, if complete independency is to be assured, no more than

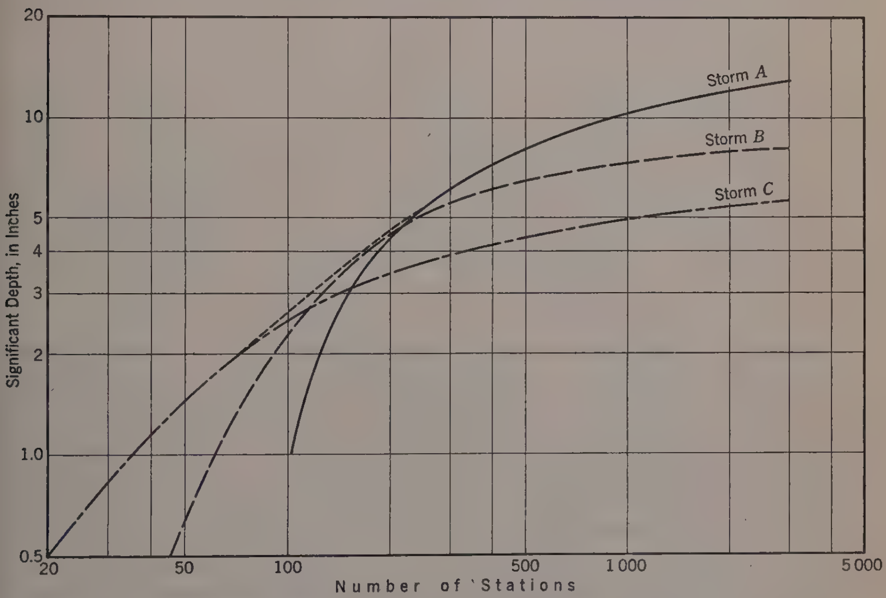


FIG. 5.—MAXIMUM ALLOWABLE NUMBER OF STATIONS TO RETAIN COMPLETE INDEPENDENCE BETWEEN STATION RECORDS

20 pegs (stations) can be utilized in record combination if 0.5 in. is considered; no more than 36 if 1.0 in. or more is taken into account; and no more than 300 if a depth of 6 in. or more is to make up the significant record.

Again, it is necessary to call attention to the artificiality of the synthesized rainfall region and the limitations of the assumptions under which the demonstration has been made. One can draw from it, however, certain conclusions

regarding the station-year method of determining the frequency of rainfall recorded at a point:

(1) The records of two or more stations cannot be combined appropriately if they are spaced more closely than the diameter (or equivalent dimension) of the storm area of significant rainfall depth.

(2) If a number of stations are so spaced, an approach to the ultimate record for one station, representative of the area, can be accelerated by combining their records, if they are of reasonable length—say greater than 5 or 10 years. From the synthetic example presented as Table 6, it is seen that 13 in. are reached at one of 36 stations in an average time of 494 min, whereas, at the end of this time, one of 20 stations would have registered about 10.0 in., and a single station only about 5 in. under the regimen of storm visitation adopted as typical.

(3) There is no basis in the combined records of a number of stations for the determination of the areal extent of storm rainfall. Records cannot be combined unless they are so spaced that only one station falls within the extent of the individual storm. As the station can occupy an infinite number of positions within the storm pattern, its rainfall depth cannot be considered an index to depths at other points within the storm area.

(4) The apparent practical success of the station-year method lies in the fact that rainfall studies, such as that of the Miami Conservancy District, have been concerned primarily with the greater depths of rainfall; and therefore (as illustrated in Fig. 5) independency can prevail among a greater number of stations than if total storm rainfall and total storm area were considered.

As storm rainfall data are made more generally available, statistical methods such as proposed in this paper will become the basis for a much sounder evaluation of magnitude and frequency than has hitherto been possible.

CHARLES F. RUFF,[31] M. AM. SOC. C. E. (by letter).[31a]—The principal value of this paper is in the emphasis placed on the probable errors of frequency computations by the station-year method, and in the incidental light it throws on all computations of flood and rainfall frequency. The map, Fig. 2, shows standard errors of 30% to more than 50% for most of the 100-yr rainfall frequencies determined in the Miami study. These values are not corrected for dependence between stations, which would, presumably, increase the range of the statistical error. Neither do they include the hydrologic uncertainty caused by the assumption that the 2° quadrangles are homogeneous in their rainfall characteristics. As the author states, this is not strictly true.

Because of a general realization of the uncertainty of frequency computations, other methods of estimating maximum floods, such as storm transposition and studies of the worst possible meteorological conditions, have come into use for the design of spillways and other purposes. Such methods, however, give no measure of the frequency of occurrence of the flood derived, except that

[31] Engr., Federal Power Comm., Washington, D. C.

[31a] Received by the Secretary February 17, 1941.

the number of coincidences assumed to occur to produce it indicate that it would be very rare.

Mrs. Clarke-Hafstad has not enlarged on the reason and necessity for determining rainfall and flood frequency. It is essential to a determination of the economics of a flood-control project, unsatisfactory as the results may be. The value of any flood-control works will depend on the number and size of future floods that it will prevent or reduce. Thus, the estimate of value, or "benefits" as it is termed, involves errors of the same order as occur in the frequency determinations. When the benefits far exceed the cost of the control works, such errors are not significant; but the writer has seen projects recommended in which the benefits were said to exceed the cost by as little as 10%. If a probable error of 30% in the frequency on which these benefits were computed exists, such a statement becomes misleading. The probable benefits would be roughly from 80% to 140% of the cost. It follows logically that in such a case the probable error of the frequency becomes critical in deciding whether the project is economically sound and should be built or not. The expenditure for such a project is not like the purchase of an automobile, but resembles more the purchase of a majority of the chances on a car which is to be raffled. The extent of the "gamble" involved is indicated by the standard error of the frequency determination.

Mrs. Clarke-Hafstad's paper deals only with rainfall frequency. For small works such as contour plowing and terracing, the rainfall can probably be used directly; but for larger areas, the methods used to transform the rainfall into flood runoff involve the assumption of values for percentage of runoff or infiltration rates. These values vary from time to time and constitute a series of independent events that combine with the rainfall to produce the flood. Thus, the flood from a 50-yr rainfall is not necessarily a 50-yr flood. The writer feels that comment on this phase of the problem would be helpful.

One of the most serious defects of the station-year method is the lack of areal significance, as mentioned by the author. For a watershed containing several stations the simultaneous rainfall at all these stations is needed to estimate the flood. If there were absolute dependence between stations, the 50-yr rain would occur at all of them at the same time. If they were entirely independent, however, such an occurrence would have a very long frequency, of the order of $50^n$ years, in which $n$ is the number of stations. The writer wonders if Mrs. Clarke-Hafstad's measure of dependence between stations would permit estimating this frequency, or preferably the frequency and amount of rain at surrounding stations when a given station had its 50-yr rainfall.

The writer is familiar with a project in which the 50-yr rainfall was estimated for each of a group of stations in a watershed, of which nearly all had less than 20 years of record. This 50-yr rainfall was then assumed to occur at all the stations simultaneously and translated into a flood. The enormous damage done by this flood was then divided by the original 50 years of the rainfall frequency at one station as a part of the calculation of annual benefits. The writer cannot but feel that such a computation contains a very large

probable error. This example illustrates the practical nature of the problems with which Mrs. Clarke-Hafstad's paper deals, and the importance, not only of reliable frequency determinations, but of a measure of their reliability.

The writer is in agreement with the author's statement that new projects of rainfall study should deal with storms rather than station rainfall readings. The latter depend on both the size of the storm as a whole and the location of the gage with respect to the storm, and are thus more complex quantities than the storm itself. Once the frequency of the storms of a region is determined, the frequency of rainfall at either a single station or the average over a given area can be computed.

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DISCUSSIONS

# CAVITATION IN OUTLET CONDUITS OF HIGH DAMS

## Discussion

BY JEROME FEE, ASSOC. M. AM. SOC. C. E.

JEROME FEE,[9] ASSOC. M. AM. SOC. C. E. (by letter).[9a]—A subject of great practical importance is presented in this paper. It deals with a difficult problem in physics, challenging modern devices of experimental technique. The authors have made available a valuable record of experience with cavitation phenomena, both at Madden Dam and from their extensive model tests.

The writer wishes to discuss the analytical methods whereby the authors demonstrate: (1) The occurrence of high local velocities; (2) the production of high intensity pressures; and (3) the percentage relation between impact forces on water and rigid surfaces, respectively.

Eq. 1 is based on the assumption that the "slug" of water travels with constant kinetic energy. In the following development, the writer assumes, instead, that the momentum remains constant. Using the same notation, the momentum of a lamina of volume $y\,dx$, having a velocity $v$, is (compare Eq. 1):

$$M = w\,(y^2\,dx)\,\frac{v}{g} = w\left(\frac{x\,d}{a}\right)^2 \frac{v}{g}\,dx \qquad (14)$$

Therefore, the total momentum of each of the slugs (a regrettable term applied to anything with the properties of a fluid) is (compare Eq. 2):

$$w\,b\,d^2\,\frac{v_0}{g} = \int_{x_1}^{x_2} w\left(\frac{x\,d}{a}\right)^2 \frac{v}{g}\,dx \qquad (15)$$

Also, from Eq. 3b, $v = v_1\left(\frac{x_1}{x}\right)^2$. Therefore, $v_0\,b = \frac{v_1\,x_1^2}{a^2}\int_{x_1}^{x_2} dx = \frac{v_1\,x_1^2}{a^2} \times (x_2 - x_1)$; or:

$$v_1 = \frac{v_0\,b\,a^2}{x_1^2\,(x_2 - x_1)} \qquad (16)$$

NOTE.—This paper by Harold A. Thomas, M. Am. Soc. C. E., and Emil P. Schuleen, Assoc. M. Am. Soc. C. E., was published in November, 1940, *Proceedings*.

[9] Associate Engr., Federal Power Comm., Washington, D. C.

[9a] Received by the Secretary January 11, 1941.

Eq. 16 should now be compared with Eq. 4, which may be written

$$v_1 = v_0 \sqrt{\frac{b\, a^2\, x_2}{x_1^3\,(x_2 - x_1)}} \qquad (17)$$

It is clear that Eqs. 16 and 4 are incompatible. Hence, if the kinetic energy is constant, as the authors assume, the momentum must change; but this means that some external force is acting of which the authors have taken no account. Their entire argument must therefore be regarded as unsound. Of course, it has no bearing on the question to say that the authors' conclusion is correct. The occurrence of high velocities in connection with cavitation is known from other evidence; but it might well be doubted if it rested on the analysis presented in this paper.

What would be the fate of this slug of water, for example, if the pyramidal section of the narrowing passage, shown in Fig. 5, had suddenly ceased to taper at a certain point, and had continued, instead, as a constant rectangular section of indefinite length? The writer can only imagine that the slug of water would tear itself to pieces in an effort to conform to the assumption that the kinetic energy is constant.

The neglect of viscosity forces is a serious defect of another character. With diminishing volumetric dimensions, approaching the apex of the pyramid in Fig. 5, the viscosity forces would become an important factor.

Referring next to Table 3, there are two very different phenomena that should be distinguished. Case (1) is illustrated by a steady stream of water, issuing from a nozzle and striking a plate normal to the direction of flow. The distinguishing feature is that the flow, although non-uniform, is steady and continuous. Case (2) is represented by the impact of a column of water flowing in a confined space, such as the inside of a pipe, and being suddenly stopped, as by the closure of a gate. In this case, the flow is neither steady nor continuous. From the instant that impact begins, ordinary flow ceases and the water particles merely oscillate about an equilibrium position, while a wave of pressure travels through the liquid.

The authors apparently have no difficulty in transferring quantitative results from one case to the other. In fact, they seem to make little distinction between them. Presumably, they have the first case in mind when they restrict the water particles to those "adjacent to the contact surface and near to the center of the contact area" (see heading "Nature and Causes of Cavitation"). Their conclusions, however, are expressed in general terms: Water moving with the velocities shown in Col. 9 of Table 3 will produce the pressure shown in Col. 1, on striking a rigid plane surface normal to its path.

The writer believes that the maximum pressure usually will be different in the two cases, for the same velocity; and that the pressure in Case (2) is the upper limit in Case (1), which may be approached but not exceeded, even in the central region of the contact area.

Case (2) is of particular interest to hydraulic engineers from its connection with water hammer. The problem is readily transformed into that of a still body of water in a pipe, which is suddenly struck by a plane surface, such as

the end of a piston. An account of tests made under such conditions was given in 1929 by H. Diederichs and W. D. Pomeroy.[10]

Although the results have long been known, for moderate velocities, from the theory of sound, it is believed that the following analysis, suggested by the method of Messrs. Diederichs and Pomeroy, is somewhat simpler than other methods of development that have appeared. Let: $A$ = area of piston, in square feet; $v$ = velocity of piston, in feet per second; $p$ = pressure against piston, in pounds per square foot; $m$ = density of water, at 62.4 lb per cu ft; $g$ = acceleration of gravity (32.2 ft per sec per sec); $C$ = velocity of sound, in feet per second; $W$ = work, in foot-pounds; and $K$ = modulus of elasticity for water (43,200,000 lb per sq ft). For moderate velocities, this modulus will serve for both adiabatic and isothermal compression.

The piston is originally at rest at the end of a pipe full of water; the static pressure is negligible. Suddenly, the piston starts to move with constant velocity, $v$. (Actually, some finite time, however short, is required for acceleration. This has only a transient effect.) A pressure equal to $p$ is produced in the layer of water adjacent to the piston, and a pressure increase travels down the pipe with the speed of sound. After $t$ seconds, the wave front has covered a distance, $C\,t$ feet, and a column of water of length $C\,t$ is now under pressure, $p$, and is moving at a velocity $v$. The work done by the piston during time, $t$, is: $W = A\,p\,v\,t$. This work is expended in two ways: (*a*) In increasing the kinetic energy of a water column of length $C\,t$ by changing its velocity from 0 to $v$; and (*b*) by increasing the potential energy of the same column, owing to the increase of pressure from 0 to $p$. This means that

$$W = A\,p\,v\,t = \frac{1}{2} A\,C\,t \frac{m}{g} v^2 + \frac{1}{2} A\,C\,t \frac{p^2}{K} \qquad (18)$$

or

$$\frac{C}{2\,K} p^2 - v\,p + \frac{C\,m}{2\,g} v^2 = 0 \qquad (19)$$

From the theory of sound

$$C = \sqrt{\frac{g\,K}{m}} \qquad (20)$$

which, substituted in Eq. 19, gives

$$\frac{1}{2}\sqrt{\frac{g}{m\,K}}\, p^2 - v\,p + \frac{1}{2}\sqrt{\frac{m\,K}{g}}\, v^2 = 0 \qquad (21)$$

Multiplying by $2\sqrt{\frac{m\,K}{g}}$ gives

$$p^2 - 2\,v\sqrt{\frac{m\,K}{g}}\, p + v^2 \frac{m\,K}{g} = 0 \qquad (22)$$

or $\left(p - v\sqrt{\frac{m\,K}{g}}\right)^2 = 0$; whence

$$p = \sqrt{\frac{m\,K}{g}}\, v \qquad (23)$$

[10] "The Occurrence and Elimination of Surge or Oscillating Pressures in Discharge Lines from Reciprocating Pumps," by H. Diederichs and W. D. Pomeroy, *Transactions*, A. S. M. E., Vol. 51, 1929, p. 9.

The foregoing shows that the pressure $p$ is proportional to the velocity $v$.

Substituting numerical values, $p = \frac{1}{144}\sqrt{\frac{62.4 \times 43{,}200{,}000}{32.2}}\, v = 63.5\, v.$

The values of $p$ for various values of $v$ are shown in Fig. 14 which shows also the authors' values. The two curves agree very well to about 205 ft per sec, but for higher velocities there is a marked difference.

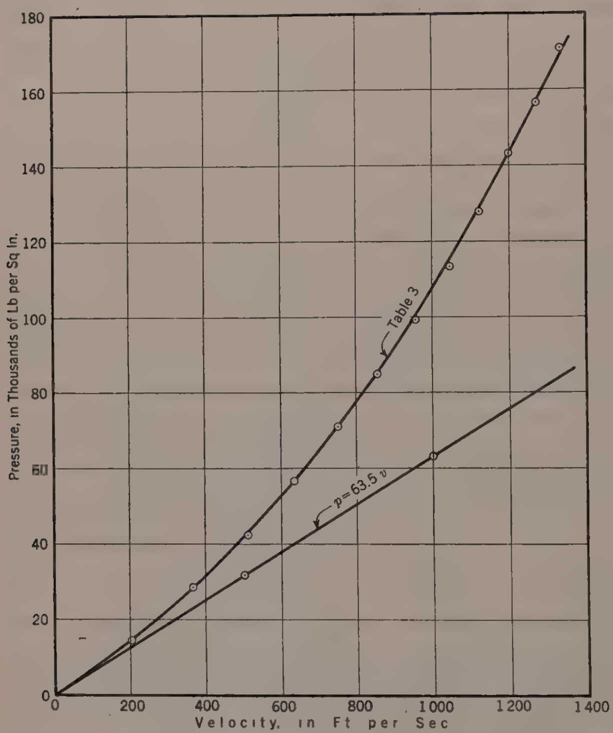


FIG. 14

Although it is interesting to have the data of Table 3 for excessively high velocities, it should be recognized that the authors present no quantitative evidence to demonstrate that velocities of this order do occur, in connection with cavitation. The maximum pressure of Table 3 is about ten times the maximum internal pressure in a 12-in. gun. Under such extreme conditions, the usual theory of sound no longer applies. Indeed, tests have shown that the sound from such a gun starts with about twice its ordinary velocity, but drops to normal in the short space of 300 ft or 400 ft. Returning to Table 3, it is not surprizing to find that the pressures determined by the theory of sound are too low.

Following Table 3, the authors develop a numerical ratio between the pressure of impact on water, and that on a "rigid" surface, illustrating their remarks by Fig. 6. The writer feels certain that the analysis is unsound.

The conclusion (see paragraph following Eq. 6) that "the velocity of particles at the contact surface is one half the initial velocity of the slug of water" is not merely incorrect, as to the magnitude of this velocity. It is based on an idea that is wholly unreal. There are no "particles at the contact surface" that can be distinguished in this manner from other particles, and whose velocity is susceptible of measurement or analysis.

Eq. 6, which the authors intend as a proof of their proposition, is nothing more than an arbitrary statement that assumes, to begin with, that the impinging particles, and those impinged upon, move during a finite interval of time with a common velocity $v_c$, which is constant. In other words, the impact between these particles is assumed to be totally inelastic. It might be well to mention that when a velocity is constant during a differential time interval it must necessarily be constant during a finite time.

In discussing these three topics, the writer is conscious that they are not indispensable to the main topic. In so far, however, as they may be subject to correction, it is important that they should be clarified.

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DISCUSSIONS

# FORT PECK SLIDE

## Discussion

BY JACOB FELD, M. AM. SOC. C. E.

JACOB FELD,[10] M. AM. SOC. C. E. (by letter).[10a]—Much more can be learned from a complete description of a single failure than from many descriptions of the design and construction of stable structures. Yet, seldom is a failure made the subject of a paper. From that viewpoint, this paper is a welcome addition to the store of knowledge on the subject of soils.

The fact of the failure is common knowledge; the causes can be deduced only from a careful and complete study of the conditions existing before and after the failure. This paper gives a complete description of the tests performed on the materials used in the dam, before and after the failure. Since the tests definitely indicated strengths in excess of the computed stresses, the conclusion is drawn that sliding (or shearing failure) along subsurface bentonite seams in the "firm shale" caused the failure. Bentonite is a treacherous material and the conclusion may well be true.

However, there are several other suspicious conditions shown by the data presented by Mr. Middlebrooks. Failure occurred near Station 15, nowhere near the location of maximum loading, or maximum computed stress on the firm shale layer. Yet, there are two phenomena that occur only near Station 15, which might be a clue to the cause of the failure: (1) The bottom of the sheet piling between stations 13 and 16 is below the "top of firm shale"; and (2) the full height of fill from the original grade of 2,050 to top of dam 2,250 occurs from Station 12 to Station 16, with soil cover over the shale only 20 ft thick. These values refer to the profile along the cutoff wall, as shown in Fig. 1. Both of these facts are definite indications of possible trouble when compared to the conditions along the remainder of the profile, where the sheet piling stopped in the region designated "except firm shale" and the cushion of natural soil between the fill and the firm shale varied from 20 ft at Station 16 to 170 ft maximum at Station 64 and to a second minimum of 70 ft at Station 84.

NOTE.—This paper by T. A. Middlebrooks, Assoc. M. Am. Soc. C. E., was published in December, 1940, *Proceedings*.

[10] Cons. Engr., New York, N. Y.

[10a] Received by the Secretary February 18, 1941.

The penetration of the sheet piling into firm shale rock makes one question whether the boring indication of firm rock is correct. If the rock could be cut by sheet piling driven through 10 ft of sandy loam and gravelly sand, and through more than 10 ft of rotten shale, then the bentonite seams that were discovered later must have been expected. The seams were probably under pressure and the firm shale was in thin layers.

The large variation in natural soil cushion always causes trouble. That is known in all foundation design and frequently has been explained as the cause for differential settlements in buildings. The natural soil was certainly less consolidated (further from the critical density) than the prepared mixture in either the core or the shells. The large variation in depth of subsidence at stations 20 and 60 is much less apparent if the total height of soil above firm shale is considered. Although the heights of fills at these two stations were the same, the total depths of rock below the ultimate top of fill are 250 ft and 370 ft, of which 230 ft is the additional fill. At the fill height of 150 ft, the total depths to rock were 170 ft and 290 ft; the corresponding subsidences (as given in Fig. 7) are 3 ft and 6 ft.

The tests of the shell materials show a high shear strength; this corresponds to a small value of shear strain at failure. The writer believes that both the core and the shells were too rigid to take up the differential subsidence or settlement without failure. Once the shear cracks had developed in the body of the fill, lateral pressure against the upstream shell, founded as it was on a water-bearing seamy rock, caused outward movement.

The effect of the surcharge load on the liquid bentonite in the seams was to increase the uplift pressure on the base of the fill. The tendency for the pressure to be relieved by escape of the liquid was nullified by the fact that bentonite does not give up its water easily and by the compression of the seams under the added load. The seamy shale might have been strong enough when not compressed and also when compressed with a sufficiently large layer of porous cover. The high hydrostatic pressure in the shale was noted in the core holes (see Table 1 and supporting text). The actual loading on the shale near Station 15 was much more than the weight of the material directly above it because of the lower subsidence at this station as compared to the remainder of the fill. Such a condition explains the findings for a hole in the fill near Station 14 + 75, down to natural surface; it is similar to the open crack which develops in a wall founded on varying depths of unconsolidated soil or in a wall founded partly on rock and partly on soil.

In the opinion of the writer, this failure is another example to prove the correctness of the rule that unequal settlement is more to be guarded against than overload. Excessive settlement did not contribute to the failure—quite the contrary. Lack of settlement due to the thin cushion, and probably also the high resistance of the relatively sudden slope in the right-hand bank, caused internal failure, and the earth pressure on the shell did the rest. To complete the paper, it is hoped that Mr. Middlebrooks will define "factor of safety" in the closing discussion.

To follow customary practice in structures, the dam should have been built in one of two ways: (1) Vertical cleavage joints, with proper shear dowels, at

about stations 20 and 85; and (2) construction of the dam between stations 20 and 85 first, closing the ends after the consolidation and subsidence were substantially complete. A combination of the two methods can be developed.

The reconstructed area that failed will not be exposed to the same transfer of load, nor the same shearing failure, since the main part has substantially reached its final position. In the writer's opinion, the failure was caused by the "erection" stresses due to differential vertical shrinkage, and, once that state has been overcome, the structure as designed by either the elastic theory method or the static slide method is stable. The paper is additionally valuable in outlining in so clear a fashion all the factors requiring investigation in the design of large earth dams.

Correction for *Transactions:* December, 1940, *Proceedings*, p. 1745, line 1, change Fig. 9 to Fig. 10.

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DISCUSSIONS

# PLASTIC THEORY OF REINFORCED CONCRETE DESIGN

## Discussion

By Messrs. R. W. Stewart, George C. Ernst, Homer M. Hadley, and Robert W. Abbett

R. W. Stewart,[40] M. Am. Soc. C. E. (by letter).[40a]—Many tests cited by the author, and other published tests not cited,[41] show that the concrete stress in a reinforced concrete beam is substantially lower at rupture than the value computed by the straight-line stress distribution assumption that is used almost universally for design purposes. This proves that the tensile steel is the weak partner and that by increasing the steel percentage a beam of given thickness can be made suitable for carrying substantially heavier loads. The author makes this situation very clear and proposes a method of analysis which he demonstrates to give accurate results. This method eliminates the modular ratio $n$ that is recognized as an arbitrary factor if the stress-strain ratio for concrete does not plot as a straight line or some known mathematical curve. However, factor $n$ can be eliminated only by the introduction of some other arbitrary device, and the author has selected the rectangular stress block, which he describes.

Flexure formulas for ultimate loads have been published heretofore on the assumption that the stress deformation curve for the concrete is a parabola.[42] The author's investigation shows, however, that a parabola of higher order than the second degree must be used to represent the stress distribution in the concrete in order to compute with accuracy the ultimate resisting moment of a beam. Table 9($a$) indicates a sequence of beam sections, each of which has the proper amount of tensile steel to fix its neutral axis at the same percentage of its depth as the beam analyzed by the author and illustrated by Fig. 2. Item 4 shows the relative moment-resisting capacity of each of the beam sec-

Note.—This paper by Charles S. Whitney, M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1941, by Messrs. L. E. Grinter, and Basil Sourochnikoff.

[40] Engr. of Bridge and Structural Design, City of Los Angeles, Los Angeles, Calif.

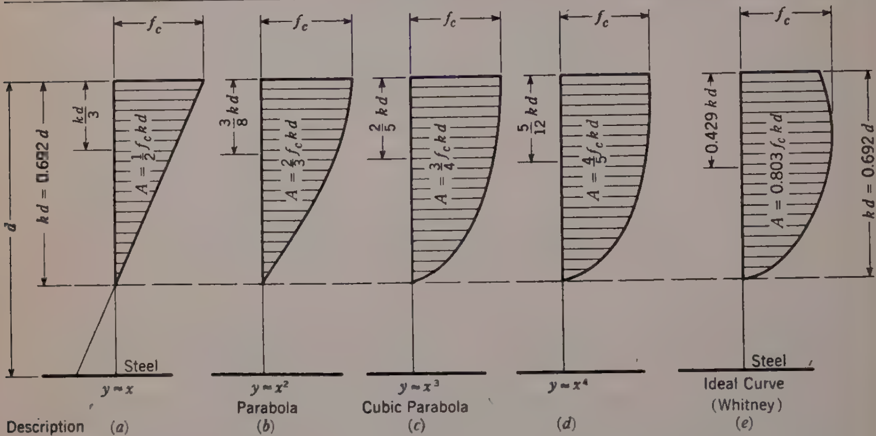
[40a] Received by the Secretary January 28, 1941.

[41] "Flexural Resistance of Shallow Concrete Beams," by Conde B. McCullough, M. Am. Soc. C. E., *Engineering News-Record*, September 19, 1935, p. 406.

[42] "Concrete Engineers' Handbook," by George A. Hool and Nathan C. Johnson, McGraw-Hill Book Co., Inc., New York, N. Y., 13th printing, p. 279.

tions. It is to be noted that the ideal curve for concrete stress distribution, presented by the author, will permit a beam, if sufficiently reinforced, to carry 47% more load than if designed by the usual straight-line formula. It may be

TABLE 9.—COMPARISON OF SLAB DESIGNS
($f_s$ = 18,000; $f_c$ = 1,000; Slab Thickness = $d$ + 2 In.; $d$ = 10 In.)




| No. | Description | Straight line (a) | Parabola (b) | Cubic parabola (c) | Fourth-degree parabola (d) | Ideal curve (Whitney) (e) |
|---|---|---|---|---|---|---|
| | (a) PHYSICAL CHARACTERISTICS; UNIFORM VALUE, $k$ = 0.692 | | | | | |
| 1 | $C/(f_c\, k\, d)$ | 0.500 | 0.667 | 0.750 | 0.800 | 0.803 |
| 2 | $j\, d$, in inches | 7.69 | 7.40 | 7.23 | 7.11 | 7.03 |
| 3 | $M/(f_c\, d^2)$ | 0.266 | 0.341 | 0.375 | 0.394 | 0.391 |
| 4 | Ratio | 1.0 | 1.28 | 1.41 | 1.48 | 1.47 |
| 5 | $d$, in inches | 10.0 | 10.0 | 10.0 | 10.0 | 10.0 |
| 6 | $k$ | 0.692 | 0.692 | 0.692 | 0.692 | 0.692 |
| | (b) COST COMPARISONS; MOMENT, IN ALL CASES, 26,600 FT-LB PER FT OF WIDTH; $d$ VARIABLE | | | | | |
| 7 | Slab thickness, in inches | 12.0 | 10.83 | 10.42 | 10.22 | 10.25 |
| 8 | $j\, d$, in inches | 7.69 | 6.54 | 6.09 | 5.85 | 5.80 |
| 9 | Area of steel, $A_s$, in sq inches | 2.31 | 2.72 | 2.92 | 3.04 | 3.06 |
| | Steel Reinforcement: | | | | | |
| 10 | Ratio | 0.0192 | 0.0227 | 0.0243 | 0.0253 | 0.0255 |
| 11 | Weight of steel in lb per ft of beam | 7.85 | 9.25 | 9.93 | 10.33 | 10.40 |
| | Dollar Costs: | | | | | |
| 12 | Steel (at 5 cents per lb) per ft of beam | 0.392 | 0.462 | 0.496 | 0.516 | 0.520 |
| 13 | Concrete (at $16 per cu yd) per ft of beam | 0.593 | 0.535 | 0.515 | 0.505 | 0.506 |
| 14 | Totals, per ft (concrete at $16 per cu yd) | 0.985 | 0.997 | 1.011 | 1.021 | 1.026 |
| 12 | Steel (at 5 cents per lb) per ft of beam | 0.392 | 0.462 | 0.496 | 0.516 | 0.520 |
| 15 | Concrete (at $24 per cu yd) per ft of beam | 0.889 | 0.802 | 0.772 | 0.757 | 0.759 |
| 16 | Totals, per ft (concrete at $24 per cu yd) | 1.281 | 1.264 | 1.268 | 1.273 | 1.279 |

noted also that using the fourth-degree parabola will give very nearly the same result. The cubic parabola, with a small error on the side of safety, appears to the writer to be a suitable curve to use. It is pictorially superior to a rec-

tangular stress block in representing the actual condition of fiber stress and involves less departure from established procedure.

A comparison of costs of slabs designed by the proposed method and by the straight-line formula was not included in the paper. Such data are presented in Table 9 (*b*), which shows the make-up of a series of slabs, each one designed to resist the same bending moment using the same fiber stresses. The differences in slab thicknesses and percentage of reinforcing steel are due to the different assumptions as to stress distribution as shown by Table 9 (*a*). It is noted that, although the author's proposed concrete stress distribution and the higher order of parabolic distributions show saving in concrete, the increased cost of the additional steel required will more than offset this saving so that some special inducement to use thin slabs is required in order to justify, economically, the abandoning of the straight-line theory of concrete stress distribution. The cost of forms is properly excluded from the cost of concrete in this comparison, so it is seen that, unless the unit cost of the concrete is considerably higher than the usual bid prices, the straight-line stress distribution formula will give the most economical result.

It may be noted also that Fig. 1 indicates that the stress-strain graph for the tests of concrete reported appears as a straight line up to a unit stress of 2,000 lb per sq in.

George C. Ernst,[43] Assoc. M. Am. Soc. C. E. (by letter).[43a]—The rational approach to the design of reinforced concrete members, as presented in this paper, has interested the writer since its introduction in 1937.[5] The straightforwardness shown in the concept of the action of the two combined materials is refreshing and should be welcomed by engineers engaged in the structural field.

It is important to note in Fig. 12 that the results of the series of tests by F. G. Thomas,[30] in which high-strength mixtures were used, do not conform to the proposed formulas although the tests of Bach and Graf[24] with a medium-strength mixture show good agreement. The Thomas tests on specimens of low-strength concrete also agree well with the formulas as shown in Table 10, but the author has suggested that the lack of agreement shown in Fig. 12 for the higher strength concrete appeared to be due to the use of enlarged ends on the specimens and slipping of the compression steel. (The use of the minimum concrete strength in Table 10 is consistent with the author's suggestion under "Method of Design and Factor of Safety." Manipulations with average, minimum, nominal, or maximum concrete or steel strengths, however, do not affect the relative positions of the groups.) The design of the ends of the specimens in the low-strength group, however, was the same as for the high-

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[43] Asst. Prof., Civ. Eng., Univ. of Maryland, College Park, Md.

[43a] Received by the Secretary January 30, 1941.

[5] "Design of Reinforced Concrete Members Under Flexure or Combined Flexure and Direct Compression," by Charles S. Whitney, *Journal*, Am. Concrete Inst., March–April, 1937, p. 483.

[30] "The Strength and Deformation of Reinforced Concrete Columns Under Combined Direct Stress and Bending," by F. G. Thomas, *Studies in Reinforced Concrete No. VI*, Building Research Station, London; see also, *Concrete and Constructional Engineering*, March, 1938, p. 165.

[24] "Tests of Reinforced and Unreinforced Concrete Columns Under Axial and Eccentric Load," by C. Bach and O. Graf, *Forschungsarbeiten auf dem Gebiete des Ingenieurwesens*, Vols. 166 to 169.

strength group. It is apparent also that the lack of agreement is just as evident for the larger eccentricities where the effect of the enlarged ends and slipping of the compression steel are virtually negligible. It is inconceivable to the writer that the design of the eccentrically loaded specimens affected the

TABLE 10.—Comparison of Actual and Computed Ultimate Loads for Columns Tested Under Axial and Eccentric Loads

| Item No. | Ratio, $\frac{e}{t}$ | Tests averaged | Ultimate Loads, in Pounds | | Eq. No. | Ratio, $\frac{\text{Col. 3}}{\text{Col. 4}}$ |
|---|---|---|---|---|---|---|
| | | | Actual | Computed[a] | | |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Series I: $p$ = 0.0245; $p'$ = 0.0045; $f_c'$ (average) = 1,480; $f_c'$ (minimum) = 1,370; and $f_s$ (average) = 45,600 | | | | | | |
| 1 | 0 | 2 | 85,300 | 77,200 | 21 | 1.10 |
| 2 | 0[b] | 4 | 105,600 | 77,200 | 21 | 1.37 |
| 3 | 0.143 | 2 | 54,450 | 51,100 | 21 | 1.06 |
| 4 | 0.357 | 2 | 41,700 | 34,100 | 21 | 1.22 |
| 5 | 0.715 | 2 | 22,900 | 21,900 | 21 | 1.05 |
| Series II: $p$ = 0.0080; $p'$ = 0.0080; $f_c'$ (average) = 1,960; $f_c'$ (minimum) = 1,630; and $f_s$ (average) = 41,900 | | | | | | |
| 6 | 0 | 4 | 113,800 | 100,900 | 21 | 1.13 |
| 7 | 0[b] | 2 | 113,900 | 100,900 | 21 | 1.13 |
| 8 | 0.071 | 2 | 86,500 | 81,400 | 21 | 1.06 |
| 9 | 0.214 | 2 | 66,100 | 59,000 | 21 | 1.12 |
| 10 | 0.571 | 2 | 30,300 | 33,400 | 24*b* | 0.91 |
| 11 | 0.857 | 2 | 18,500 | 20,700 | 24*b* | 0.90 |
| Series III: $p$ = 0.0080; $p'$ = 0.0080; $f_c'$ (average) = 4,640; $f_c'$ (minimum) = 4,370; and $f_s$ (average) = 43,700 | | | | | | |
| 12 | 0 | 3 | 183,200 | 216,400 | 21 | 0.85 |
| 13 | 0 | 2 | 280,500 | 216,400 | 21 | 1.31[c] |
| 14 | 0[b] | 2 | 174,200 | 216,400 | 21 | 0.81 |
| 15 | 0.071 | 2 | 157,800 | 172,300 | 21 | 0.92 |
| 16 | 0.214 | 2 | 112,400 | 123,100 | 21 | 0.91 |
| 17 | 0.429 | 2 | 58,600 | 77,700 | 24*b* | 0.75 |
| 18 | 0.571 | 3 | 43,700 | 51,900 | 24*b* | 0.84 |
| 19 | 0.857 | 3 | 23,100 | 25,800 | 24*b* | 0.90 |

[a] Minimum $f_c'$ and average $f_s$ used in each group. [b] Tested with fixed ends. All others tested with knife-edge end bearings. [c] The cylinder strength (converted) exceeded the average by 17.5%.

results to any noticeable degree unless it is due to the smallness of the cross section (about one sixth the area of the Bach and Graf test columns) when used with the higher strength mixtures. In this connection it is to be noted that the 10-in. square test columns of Richart and Olson had an area of only about one third of the Bach and Graf specimens. Although Messrs. Richart and Olson suggest the possibility that about a 15% reduction in the capacity of "hinged-ended" columns exists as compared to "flat-ended" columns, nevertheless the tests of Thomas fail to confirm this belief when enlarged ends are used. Since the effect of the enlarged ends should be essentially the same for "hinged" and "fixed" ended specimens, the tests by Professor Thomas are definitely inconclusive in this matter (see Table 10). With regard to the five points plotted in Fig. 12 on the ordinate $e$ = 0, the lowest value and the next

to the highest value are for "fixed-ended" conditions, whereas the remaining three are "hinged-ended." Two other "hinged-ended" columns, not shown, would fall above the entire diagram although the concrete strengths of these two were only 17.5% above the average of the group. Furthermore, it is difficult to attribute the lack of strength in the tests of Messrs. Richart, Olson, and Thomas to hinged ends if the Bach and Graf tests are considered as hinged-ended for the eccentrically loaded conditions.

All evidence points to a reduction in relative capacity due to the use of the higher strength mixtures, although it must be recognized also that the size of the specimens of Bach and Graf are comparable to actual construction, whereas those of Thomas are somewhat small. It would seem best to the writer to place a limit on the concrete strength at approximately 3,000 lb per sq in. for the proposed formulas until more conclusive tests have been conducted using the higher strength mixtures under combined bending and direct stress.

Although Table 6(*a*) is entitled "Rapid and Long-Time Tests," the frames subjected to long-time loading by Messrs. Richart and Olson are not tabulated and it would prove of value if the author would supply the missing information for the purpose of showing that the effect of sustained loading is negligible. The writer believes, also, that much of the value of Table 6(*a*) is lost unless the frame details are described briefly to show the effect of various sizes of fillets and chamfers. Such details raise a question as to the position of the critical section, and in so far as the writer has been able to determine it occurs, for these frames, just inside the fillets, whether circular or straight. Chamfering the back corner may bring the critical section into the corner if the effective

TABLE 11.—COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS FOR RECTANGULAR FRAME TESTS

(Clear Opening, 4 Ft by 2 Ft; Third-Point Loading; Low-Strength Concrete Group; $f_c' = 1{,}510$; $f_s = 56{,}800$ at the Yield Point; Steel at Midspan = 1.47 %; $p' = 0$)

| No.[a] | Ratio, $\frac{e}{t}$ | ULTIMATE LOADS (LB) | | Ratio, $\frac{\text{Col. 2}}{\text{Col. 3}}$ |
|---|---|---|---|---|
| | | Actual | Computed | |
| | (1) | (2) | (3) | (4) |
| 1[b,c] | 4.50 | 10,200 | 9,100 | 1.12 |
| 2[c,d] | 4.50 | 8,800 | 9,100 | 0.97 |
| 3[c,d,e] | 2.67 | 4,800 | 4,960 | 0.97 |
| 4[d,f] | 8.90 | 6,000 | 6,200 | 0.97 |
| 5[d,e,f,g] | 2.67 | 4,800 | 4,960 | 0.97 |

[a] One test for each item. [b] No hinges. [c] Constant 6-in. depth and 3.5-in. width. [d] Bases hinged. [e] Hinged at crown. [f] Verticals varied from 6 in. at the knee to 3 in. at each base hinge. [g] Top horizontal varied from 6 in. at the knee to 3 in. at the hinges.

section at the corner is reduced sharply, although this does not appear to be the case for frames 6 or 7.

Tests on rectangular frames made at the University of Maryland in 1940 (Table 11) show good agreement with the proposed formulas, but it must be noted that the concrete mixture was of low strength. Tests upon frames very

closely approximate actual conditions of restraint in the application of the eccentric load.

The important feature revealed by the comparison of the author's formulas with test results seems to be the need for additional information on rectangular and circular sections subjected to eccentricities from zero to pure bending and covering a wide range of concrete strengths. Until such additional tests are made it would be exceedingly difficult for the writer to justify the use of the proposed formulas without an upper limit of 3,000 lb per sq in. placed upon the concrete cylinder strength.

Corrections for *Transactions:* On page 1771, line 3 from the bottom, change "Table 6(*b*)" to "Table 6(*a*)"; and on page 1773, line 3 from the bottom, change "$d$ = 5.15" to "$d$ = 5.75."

HOMER M. HADLEY,[44] ASSOC. M. AM. SOC. C. E. (by letter).[44a]—There are several matters concerned with the fundamentals of Mr. Whitney's proposed theory of design on which it is hoped additional information will be furnished.

(1) Fig. 1, from which the proposed theory is developed, shows six curves, one of which terminates at a unit strain of 0.0036 in. per in., a second at a unit strain of 0.005 in. per in., whereas the remaining four are shown extending unbroken to the right-hand edge of the diagram. Standard 6-in. by 12-in. cylinders being used, the unit strain 0.005 in. per in. gives a total deformation of 0.06 in. The text states (see heading "The Stress-Strain Characteristics of Concrete") "the machine head was moving continuously at the rate of 0.06 in. per min." On this basis the range of stresses shown in the diagram occurred in slightly more than $1\frac{1}{2}$ min. It is most immediate, practically instant, plasticity that is disclosed.

These are highly unusual stress-strain curves for concrete cylinders. Ordinarily the attainment of maximum stress is closely followed by failure. For a few seconds while it holds together the load drops off rapidly; then the cylinder shatters and breaks into fragments. In Fig. 1, however, the cylinders do nothing of the kind. After maximum stress has been attained (that is, after maximum load has been applied) they do not fracture but simply deform with reducing unit stress. It might almost be said that they are made to deform by taking load off of them, which, of course, is an improper statement. The truth is that they are made to deform uniformly and they offer sustained but decreasing resistance. Complete details regarding these tests and such anomalous behavior are most desirable.

(2) Under the foregoing heading is the statement that

> "The effect of time on the stress-strain relation is believed to be similar to that in the case of tests on beams. It is probable, therefore, that these curves indicate the form of compressive stress distribution in a beam at ultimate load in the concrete in the zone where the unit strain exceeds about 0.002 in. per in."

[44] Regional Structural Engr., Portland Cement Assn., Seattle, Wash.

[44a] Received by the Secretary February 18, 1941.

Considering that the cylinder test is one of direct stress over a full section, whereas the beam is a case of flexure with fibers at varying distances from the neutral axis; and considering the highly unusual character of the curves in Fig. 1, the writer hopes there will be further exposition of these statements. Has it definitely been "proved" or has it definitely been assumed "that the strain in the compression side of a beam increases practically in direct proportion to the distance from the neutral axis"?

(3) When the stress distribution in a beam has been explicitly stated to be "probably" like those of the cylinder curves of Fig. 1, why should the direct declaratory form of statement subsequently be used regarding this stress distribution—for example, in the second paragraph following Eq. 3*b*? The validity of an assumption is not established simply by its being postulated and formulated.

(4) Why was the particular form of stress curve shown in Fig. 4 selected? This curve appears to be identical with the curve of Fig. 2 which is definitely stated to be based on an $\epsilon$ value of 0.003. If it is assumed that the four curves of Fig. 1 do indeed terminate at the margin of the diagram at a unit deformation of 0.008 in. per in., then the average value of $\epsilon$ of the six curves would be 0.0068—say, 0.007. If this average value of the anomalous evidence were used, as apparently it might well be, then a different stress distribution in the beam would result with a different position of the center of gravity, etc. Of all the "fibers" (so-called) in a beam, the extreme topmost one is most advantageously placed with respect to the centroid of tensile stresses for developing resistance to external forces. In view of this fact is there not an inherent improbability in the assumption that a less advantageously placed fiber with a substantially lesser lever arm carries a higher stress than the topmost fiber? In the general economy of nature things do not work in this fashion.

This matter of so-called "plastic flow" in concrete is an annoying one. There is a nucleus of truth in the term. A certain slight amount of increased deformation does occur at early ages under sustained load; but how this is to be separated and distinguished from the deformation that occurs when concrete dries out and shrinks in plain, common every-day fashion is difficult to determine. The concrete knee-frame tests under sustained load, reported by Messrs. Richart and Olson,[25] had the specimens continuously wet until placed under load and it was while under load that they dried out. So likewise continuously wet until placed under load were the specimens of the A. C. I. column tests at Lehigh University, Bethlehem, Pa., and the University of Illinois, Urbana, Ill. Plastic flow is such an easy explanation of other things. Some one forgets to put camber in a bridge span and it settles slightly. "Plastic flow" is the immediate whispered explanation. Embarrassing deflections from some other cause—insufficient reinforcement, for example—develop and—"Oh, that's plastic flow in the concrete. All the research laboratories are studying it these days"—and so on. It is worthy of note that most concrete structures, if they are any the worse for it, do not reveal that fact to any appreciable extent. In a publication of Northwestern Technological Institute of North-

[25] "Rapid and Long Time Tests on Reinforced Concrete Knee Frames," by F. E. Richart and T. A. Olson, *Journal*, Am. Concrete Inst., March–April, 1937, p. 459.

western University dated February 1, 1941, Prof. G. A. Maney,[45] M. Am. Soc. C. E., reaches the conclusion that "Concrete is not an appreciably plastic material under working stresses and the time changes, in volume and dimension, of concrete under working loads are not due to plastic flow but to shrinkage." Mere change of nomenclature, of course, does not eliminate the volume change but it does simplify matters; it does eliminate obscurantism.

The standard concrete compression-test specimen, a cylinder 6 in. in diameter and 12 in. high, is an interesting object to consider. It is necessary to have some means of comparing various mixtures of concrete made at different places and at different times; and it has come to pass that the standard cylinder is the generally accepted means of making such comparisons; but, out of the complete widespread acceptance of this cylinder and of its unit compressive strength as a basis of comparison, has developed the attribution to it of wholly unwarranted powers. Of these none is more lacking in substantial basis than the assumption that it measures, directly, the flexural strength of its concrete when that concrete is put in a beam or slab and its ultimate fiber stress is computed by the straight-line theory with some one of the various values of $n$. Just why it should be a cylinder 6 in. in diameter and 12 in. high that should do this rather than some other cylinder 6 in. in diameter and of a different height is not, on earnest consideration, apparent. In Europe concrete is tested in 6-in. cubes and has some other unit strength than it has in America. Let no one think that shape of test specimen and conditions of testing do not have a pronounced effect upon the nominal unit strength of concrete. Some years ago S. H. Woodard, M. Am. Soc. C. E., tested numerous cylinders and prisms with sheets of blotting paper, impregnated with heavy grease, introduced between the ends of the specimens and the heads of the testing machine. By so doing he assuredly did not change the inherent properties of the concrete but he eliminated conical and pyrimidal breaks completely and caused their strengths to drop markedly below those of companion specimens tested without the blotting paper.[46] The paper by H. F. Gonnerman,[47] M. Am. Soc. C. E., indicates that the height of the 6-in. diameter cylinder should be about 4 in. to have it measure, directly, the flexural strength of the Slater-Lyse and other test beams when computed by the straight-line theory.

However, when it is possible to multiply the unit strength of the standard 6 by 12 cylinder by 1.5 and thereby obtain a value that reasonably approximates the flexural unit strength of the concrete in test beams, what could be simpler than to do that? The writer is not especially "enamored" of the straight-line theory, but on the other hand it is to be recognized that it has the virtue of great simplicity. Furthermore, it is the established and practically world-wide basis for designing reinforced concrete and has been taught continuously in all engineering schools and colleges ever since there has been a standard method of design. It is not to be lightly abandoned and put aside.

[45] "'Warping' or Non-Uniform Shrinkage Over Cross-Section Found to Be Source of All Time Yield in Loaded Concrete Members," *Studies in Engineering—No. 1*, Northwestern Technological Inst. of Northwestern Univ., Evanston, Ill.

[46] *Transactions*, Am. Soc. C. E., Vol. 99 (1934), p. 1091.

[47] *Proceedings*, A. S. T. M., 1925, Pt. 2, p. 237.

Therefore, as a constructive proposal it is suggested that the ordinary cylinder values be multiplied by 1.5 to obtain flexural values conservatively less than the ultimate flexural strength, that the straight-line theory be retained, and that working stresses be based on the flexural, not on the cylinder, strength. Certainly there is need for a revision of concrete design practice to effect a more reasonable and more economical use of the material than at present obtains. Fundamentally such a revision will make it permissible for concrete structures to carry greater loads than now allowed. That is what concrete structures are for—to carry loads. To make them do that effectively, economically, and in reasonable conformity with their potential abilities means the revision of various "books of rules" and a marked increase in present maximum permissible stress values.

Robert W. Abbett,[48] M. Am. Soc. C. E. (by letter).[48a]—The theory proposed in this paper, for proportioning structural members of reinforced concrete, is based on the assumptions that: The material is plastic; stress is not proportional to strain; and, in the case of gradually applied loads, the material will fail at a stress intensity less than maximum. These conditions are known to exist when the order of magnitude of compressive stress in the concrete is near the ultimate. Therefore, the author's theory should give an accurate indication of the behavior of a reinforced concrete member at the point of impending failure, provided that the strength of the concrete is critical and that the applied loads are of long duration.

In the case of loads suddenly applied and quickly removed, the stress-strain curve for concrete is not very similar to the ideal curve in Fig. 3 in that failure usually occurs near the point of maximum stress. This is true even when the rate of load application is that usually adopted as standard for the testing of cylinders in the laboratory. Frequently, in laboratory tests of cylinders, the failure is in the nature of a sudden explosion at or near the point of maximum stress.

If the reinforcement of a beam is under-designed, presumably ultimate failure will occur because of yielding of the steel when the concrete stress is at some point below the maximum for the material. When this occurs the concrete stress is likely to fall in a region of the stress-strain curve which is a closer approximation of the properties of an ideal elastic material than those of an ideal plastic material. In this connection it should be noted that, even when the design is perfectly balanced, failure will usually occur because of yielding of the reinforcement, and that under any circumstances a compression failure of the concrete is very rare. Furthermore, when a factor of safety of 2 or more is used in the design, the working stresses in the concrete are in the elastic range of the stress-strain curve as in the case of the under-reinforced beam at the point of failure.

In view of the foregoing conditions, the writer has drawn the following conclusions with regard to the plastic theory:

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[48] Asst. Prof., Civ. Eng., Columbia Univ., New York, N. Y.

[48a] Received by the Secretary February 21, 1941.

(1) In the case of structural members for which the greater part of the load to be carried is dead load or live load permanently fixed in position, the plastic theory gives a better indication of the ultimate strength of the member than the present standard theory based on the theory of elasticity. Typical of such members are the column, footing, arch rib, and floor girder.

(2) When movable live load comprises the greater part of the total load to be carried, the plastic theory may give an erroneous indication of the strength and behavior of the member, and the elastic theory is to be preferred. Included in this class of members are most building floors, bridge decks, beams, and stringers.

(3) The plastic theory gives very little indication as to what is occurring in a member working at the average, or design, stresses when the factor of safety is 2 or more. On the other hand, the elastic theory gives results that are relatively close approximations for a large class of structures.

(4) The prediction of deflections under working conditions, although very uncertain with the elastic theory, is virtually impossible with the plastic theory.

(5) There are uncertainties in the behavior of reinforced concrete members that are ignored by both the plastic and elastic theories. These are probably the cause of greater variations than those due to the two theories mentioned. Typical of these uncertainties are the effect of tension resistance in the concrete, variation in effective moment of inertia, and stress conditions in both the concrete and steel in the vicinity of the small cracks which are always present in a stressed beam.

(6) Apparently the balance is about even when it comes to a choice of the two theories with a slight advantage in favor of the elastic theory simply because it is in agreement with the procedure standardized for other materials. However, the entire idea of "working stress" as a basis for design is questionable. There is a growing tendency to think of design in terms of useful limiting load but until this becomes universal there is no reason why the plastic theory should not be adopted for the class of structures for which it is most suitable. This has been done already in the case of columns.

Finally, the writer would like to pay his respects to the author, whose paper has contributed so much to the theory of reinforced concrete structures.

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DISCUSSIONS

# EXPERIENCES IN OPERATING A CHEMICAL-MECHANICAL SEWAGE TREATMENT PLANT

## Discussion

By Messrs. F. C. Roberts, Jr., and Rolf Eliassen

F. C. Roberts, Jr.,[4] Assoc. M. Am. Soc. C. E. (by letter).[4a]—In this very complete paper, Mr. Schroepfer has outlined the various steps involved in the new plant for the Minneapolis-St. Paul Sanitary District. When all units of this plant are in service, the sewage passes through the following steps: A coarse bar screen with 6-in. openings and a bar screen with $1\frac{1}{2}$-in. openings; then from these screens the sewage flows to the grit chambers; and thence to the flocculating tanks, the settling tanks, and finally the magnetite filter units. Before flocculation, the sewage entering these basins may be dosed with chemicals to aid in sedimentation and clarification.

One of the features of this plant is the vacuum filtration and incineration of solids removed from the sewage. It is to be noted from the information presented by Mr. Schroepfer that the cost of this disposal represents approximately one half of the plant operation cost. A question arises as to the relative cost of this process over the digestion tank processes, or its necessity, in so far as construction and operation costs are concerned.

Throughout this paper it is apparent that the objective was the improvement of the pollution of the Mississippi River. The paper is presented in terms of the tonnage load of pollution delivered to the river. In most public health work engineers are not concerned with the percentage of removal of solids by a particular unit of a disposal plant. They are not interested in the percentage efficiency of the settling basin, or the secondary or tertiary processes; but they are interested in the final effluent that reaches the river. If the plant effluent is consistently of the same degree of purity, so that the river is neither a nuisance nor a public health menace, then they are satisfied. With the most complicated disposal process, the final results may not be satisfactory to the river, and a river pollution condition results. It is of no little interest

Note.—This paper by George J. Schroepfer, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*.

[4] State San. Engr., State Board of Health, Phoenix, Ariz.

[4a] Received by the Secretary January 31, 1941.

to note that this paper refers to the end product that must be met, and the plant is built back from this criterion.

This plant was not designed unit by unit for maximum flow or average flow conditions, but rather to meet the variable flow conditions that might be expected at the plant. As Mr. Schroepfer states, the period of time that chemical treatment will be needed will be about 29 days in one year. One may imagine the necessity for chemical treatment during this relatively short period. The availability of chemical treatment will furnish the operator with a tool with which to meet adequately a condition that will arise at intervals, and a uniform result may be expected at the point of plant outfall. Flexibility appears to be the cardinal virtue of this plant.

The writer has had no experience with that part of the country around Minneapolis and St. Paul. In discussing chemical treatment, it is necessary that his remarks be confined to the six or seven chemical treatment plants in Arizona in order to furnish some basis of comparison with the plant presented in the most thorough and excellent paper by Mr. Schroepfer. It was thought that it might be of value to discuss the Scott-Darcy process as it has been developed in Arizona. The development of this process has been the one major contribution that Arizona has made to the field of chemical treatment to date.

The needs within Arizona are different from those encountered elsewhere in the United States. Because of its freight rates, high temperatures, and wide variations in sewage flow due to a high rate of water usage, it was necessary to use some chemical treatment process that was flexible, and that would be available when needed and stable until needed. A chemical that might be delivered to any point with a minimum of cost and inconvenience was also a requisite.

There are four general types of chemical treatment: First, chemical treatment is used to aid in coagulation and flocculation, as provided in the Minneapolis-St. Paul plant; second, sewage is treated with certain chemicals to remove objectionable characteristics, such as the sulfur compounds that cause the most objectionable odors and the deterioration of concrete sewers; third, chemicals are used to maintain the sewage in a fresh condition until it reaches the plant; and fourth, sewage is sterilized to remove the bacteria.

The first classification of this purely arbitrary grouping is accomplished with any of the metal precipitants, such as ferric iron, alum, etc. The second process is the application of ferrous iron or iron in such form that it combines readily with sulfur. The third and fourth types are differentiated only in the degree of treatment that is used. These types of treatment make use of chlorine and its compounds of hypochlorite. The latter type satisfies the entire demand of the sewage and then leaves a "residual," or excess, for the complete sterilization of the flow.

In Arizona the problems have involved all of these types. The Scott-Darcy process has been used for the manufacture of iron in such form that it may be used for the first and second uses outlined previously. By modifying the equipment, the plant may be used for the other two purposes, although this has not been the general practice to the present time (1941). Many installa-

tions in Arizona have been made on collection and outfall sewer systems for the purpose of removing the sulfides from the field of action, to reduce odors, and to retard the corrosive action of the sulfides on concrete sewers.

It will be recalled that chlorine and some of its compounds have been used for a number of years for the aforementioned purpose. As long as the sewage is sterilized in its initial or very fresh stage, the use of chlorine has been effective. After decomposition has proceeded to a certain point, the use of chlorine is expensive and does little to remove sulfides already formed.

In line with the pioneer work of the Scott-Darcy process as established in Oklahoma, the City of Tucson, Ariz., installed the first Scott-Darcy plant in Arizona. This was for the purpose of removing sulfides already formed in the sewage, and for the addition of sufficient iron to the sewage to react with the sulfide delivered to the outfall from the many interceptors that discharge into it below the point of application. Also, a major purpose of this installation was to control odors at the disposal plant. An installation similar to the original Scott-Darcy was installed. This installation allows superchlorinated water to enter one end of a contact or reaction tank that is filled with scrap iron. After a period of contact therein, the iron and chlorine solution flows out of the far end of the tank and enters the sewage.

This installation has resulted in a lower "free sulfide content" of the sewage to a point 2 miles below the point of iron-chloride application, where the sulfides again begin to increase until the plant is reached. It is apparent that either a larger dosage of iron-chloride at the present plant or another plant is necessary at the point where the sulfides again begin to increase.

This same type of installation was made at Chandler, Glendale, Phoenix, and Wickenburg, Ariz. In each instance the control of odors was of extreme importance, and the plants have achieved this purpose. It has been expensive to operate them, yet in each instance any chemical or other treatment would have been expensive. The installation of these plants has forestalled some expensive litigation, which in at least two instances had been instituted against the cities or towns.

Phoenix has stopped the deterioration of concrete sewers with the process by the installation of some four plants throughout the system. This fact has been determined by visual inspection only. Prior to the installations, the wall thickness of some concrete sections had deteriorated 50% in four years. It is hoped that this process will extend the life of these sewers to last out the length of the bond issue, or until 1962. Chandler has stopped the odors from a septic tank. Lawsuits were pending against the city. By the addition of Scott-Darcy, these lawsuits have been dropped at least temporarily. Glendale has corrected an odor condition at this plant, and is using the iron as a precipitant after the addition of lime to form the iron hydrate. This installation has worked satisfactorily for the manufacture of deodorant and precipitant. Wickenburg has installed such a plant, but it needs further investigation to determine how effectual it will be.

During this period of development, the more modern Scott-Darcy process was developed at the Tovrea Packing Plant. It was the purpose of this installation to provide odor control and a precipitant, to have free chlorine

available for the coagulation of blood in the slaughterhouse waste, and to have available a sterilizing solution if it was found to be of use. The use of the Scott-Darcy plant for the latter two purposes has not been found to be necessary to the present time (1941). The raw waste entering the plant ranges in strength from 1,000 to 5,000 ppm of suspended solids. After flocculation and sedimentation with the use of iron chloride from the Scott-Darcy unit, the final effluent has a suspended solid range from 65 to 442 ppm. The effluent has been reduced in B.O.D. strength, with a range of from 750 to 2,200 ppm down to 113 to 300 ppm.

The difference between this Scott-Darcy plant and the other plants is that the iron-chlorine solution is recirculated through special pumps so that the iron solution may be chlorinated to that particular point desired by the operator, and the required type of iron chloride produced. The flexibility of the plant to meet changing hourly demands has proved an excellent tool in the solution of this problem. The packing plant officials, health agencies, and the neighbors are all satisfied with this plant.

The conditioning of this sewage has made available an irrigation water supply that is used on 160 acres of land for the production of forage crops to be fed to the beef cattle held at the plant. Prior to the installation of this plant, packing house waste was discharged into the Salt River to the embarrassment of the various health departments and to the despair of those living in that vicinity.

From the detail of information offered in Mr. Schroepfer's analysis of the Minneapolis-St. Paul plant, and from these casual observations, it might be concluded that chemical treatment is vitally necessary when needed. It furnishes a flexibility of operation that is difficult to attain otherwise. It may be for only a short period of time during the day, week, or month that chemical treatment might be needed. When available, it levels off a peak effectively, and improves operation.

In conclusion, the writer desires to express appreciation to J. A. Carollo and Dario Travaini, Associate Members, Am. Soc. C. E., Phil J. Martin, Jr., of Tucson, and W. J. O'Connell of San Francisco, Calif., for the generous assistance that they have provided in the preparation of this discussion.

ROLF ELIASSEN,[5] ASSOC. M. AM. SOC. C. E. (by letter).[5a]—Complete operation and cost data on the vacuum filtration and incineration of the raw sludge at the Minneapolis-St. Paul Sewage Treatment Plant are presented in this paper. Data of this nature are essential in order to obtain a better economic evaluation of the various methods of sludge treatment and disposal.

Most of the larger cities in the United States have complex problems of sludge disposal. The sludge is usually digested, followed by drying on sand beds or vacuum filters and ultimate disposal on land as fertilizer or fill. Some cities prefer to subject raw or digested sludge to vacuum filter and either dry it mechanically or incinerate it before ultimate disposal. As the author has stated, in the Minneapolis-St. Paul plant the raw sludge is conditioned with

[5] Assoc. Prof., San Eng., Coll. of Eng., New York Univ., New York, N. Y.
[5a] Received by the Secretary February 17, 1941.

chemicals, partly dewatered on vacuum filters, then dried and incinerated in multiple-hearth furnaces.

The City of New York, N. Y., is more fortunate in that it is located on the Atlantic Ocean and can utilize that body of water for disposal of its sludge. In 1940, Wellington Donaldson,[6] M. Am. Soc. C. E., described New York's method of sludge disposal. The sludge from three treatment plants is transported in sludge vessels designed for that purpose. They are three sea-going vessels of the latest design and have sludge storage capacities of 55,000 cu ft each. The sludge dumping grounds are 34 miles out to sea from the Ward's Island Sewage Treatment Plant. These grounds are far enough removed from the shore to preclude any danger of harbor or coastal pollution.

Utilizing data presented by the author and Mr. Donaldson,[6] an interesting comparison may be made between these two distinctly different methods of sludge disposal.

These comparative data for the year 1939 are as follows:

| | Minneapolis-St. Paul | New York City |
|---|---|---|
| Initial cost, in dollars | 1,240,000 | 1,500,000 |
| Total sewage flow treated, in million gal | 36,705.3 | 62,696.1 |
| Total sludge, dry tons | 37,703 | 54,088 |
| Operating and Maintenance Costs (in Dollars): | | |
| Total | 136,815.43 | 137,931.50 |
| Per million gal of sewage treated | 3.74 | 2.20 |
| Per ton of dry sludge | 3.63 | 2.55 |

In the case of Minneapolis-St. Paul, the initial cost includes the cost of constructing the filtration and incineration building and appurtenances. For New York, the cost is that of constructing the three sludge vessels. As far as one may ascertain from an analysis of the supporting data and discussions, these cost analyses are on a comparable basis.

Further economies are possible in both cases. This is evidenced by the fact that data for the first five months of 1940 at Minneapolis-St. Paul show costs of sludge disposal at $3.15 per ton of dry solids, a reduction of 13.2%. This has resulted from more efficient use of chemicals, filters, and incinerators. Even greater economies are possible in New York as means are devised for the thickening of the sludge before loading on the vessels. The 55,000 cu ft of storage capacity should be filled with the heaviest possible sludge in order that the tonnage of dry solids may be at a maximum for each trip to the dumping grounds.

[6] *Waterworks & Sewerage*, December, 1940, p. 583.

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DISCUSSIONS

# ANALYSIS OF STATICALLY INDETERMINATE TRUSSED STRUCTURES BY SUCCESSIVE APPROXIMATIONS

## Discussion

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By Francis L. Castleman, Jr., Assoc. M. Am. Soc. C. E.

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Francis L. Castleman, Jr.,[7] Assoc. M. Am. Soc. C. E. (by letter).[7a]—The term "classical method" is a misnomer when used to describe the general method to effect a solution for the internal redundants of frames, such as are shown in Figs. 1, 2, and 4. Classical methods, in such cases, are modern methods, since basically for problems of this type, a solution must always be effected by the method of deflections. Whether recourse is had in the detail of solution to least work, dummy unit loading, virtual work, etc., is a matter of individual choice; all lead to the same end. All analytical methods are based on the elastic energy of the system and can be derived from the general principles laid down for such methods.

Eqs. 3 express the gist of the matter. Once these equations are properly set up and their constants correctly determined the engineering is done. They have been known for a long time; they are still mandatory for the solution of many problems, and hence are modern in the sense that they are indispensable. Thus the writer questions if Mr. Voodhigula has simplified, in any way, the so-called classical method of analyzing the statically indeterminate structures in question. What he has really done, in the writer's opinion, and very ably, is to devise an extremely ingenious method for the detail of the solution of Eqs. 3.

After trying this method on several "pet problems" it was found to be quite helpful in some respects, but no faster than the ordinary method of iteration without the graphical interpretation such as given in Fig. 3. It is more compact and offers a visual interpretation of what is believed to happen. However, in all fairness, it must be said that perhaps complete facility was not acquired in its use. The implied visual interpretation of stress distribution is really a "second sight" as the true physical and visual interpretation is implied in Eqs. 3 and their proper development.

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Note.—This paper by O. T. Voodhigula, Jun. Am. Soc. C. E., was published in January, 1941, *Proceedings*.

[7] Assoc. Prof., Structural Eng., Vanderbilt Univ., Nashville, Tenn.

[7a] Received by the Secretary February 5, 1941.

Errors of analogy are often introduced in an analysis when operations performed are remindful of those executed in a like manner in a previous but non-similar situation. In moment distribution there is an entirely different situation and one operation in the solution of this situation involves an action the end result of which is termed "carry-over." In the situation under consideration, which by no stretch of the imagination is similar to moment distribution, a name for an operation is literally "carried over" from a non-similar situation. On reflection the writer believes the term "stress factor" would be more appropriate. Certainly, if such were the case, excuse could never be offered (by analogy) for dividing the carry-over factor by 2 (or any other value).

Mr. Voodhigula dismisses the problem of external redundants with far from the proportionate space devoted to internal redundants; yet the former is generally the problem confronted, as structures of the latter type are few (at least in American practice) as compared with the former. This would seem the most fruitful field for the method under discussion. However, a more than cursory examination of such structures leads to some very interesting observations as regards the accuracy of solution by any method. Sometimes it is quite advantageous to choose a member or members of such a structure as the redundants rather than the reactions. For example, in Fig. 8 the reactions "*a*," "*b*," and "*c*" are treated as redundants. This presupposes for the statically determinate base system the full span acting as a simple truss under the appropriate loadings. Equations can be set up similar to Eqs. 3 and reduced with proper manipulation and interpretation such that, by their use, influence lines can be constructed for each of the external redundants, and subsequently those for the individual members in the truss. The latter step can be achieved by cutting the lower chord at "*a*" and the upper chord over "*b*" and "*c*" and treating the cut members as redundants. The statically determinate base system then becomes three simple-span trusses. Comparable errors will be greatly reduced, therefore, with consequent reduction in the errors of the required redundant values. American textbooks have paid too little attention to this fact. By way of illustration, consider:

$$X = \frac{\begin{vmatrix} \delta_m' & \delta_{mn} \\ \delta_n' & \delta_{nn} \end{vmatrix}}{\begin{vmatrix} \delta_{mm} & \delta_{mn} \\ \delta_{nm} & \delta_{nn} \end{vmatrix}} = \frac{\delta_n' \, \delta_{nn} - \delta_n' \, \delta_{mn}}{\delta_{mm} \, \delta_{nn} - \delta_{mn}^2} \quad \text{(11)}$$

and with the following numerical values inserted, $X = \dfrac{50 \times 100 - 90 \times 60}{40 \times 100 - (60)^2}$ $= -1$. If the errors of determining the $\delta$-values vary by $\pm 2\%$ then, in the extreme, $X = \dfrac{51 \times 102 - 88.2 \times 58.8}{40.8 \times 102 - (58.8)^2} = 0.225$. Thus a tremendous error has been introduced from small, possibly unavoidable errors in the basic quantities. Through use of a properly chosen statically determinate base system such errors can generally be held to a minimum. Exact solutions, computing machines, methods of iteration, and even Divine Providence cannot eliminate such errors once they are present in the fundamentally basic quantities.

In the interest of rigor it would seem logical to emphasize that the theoretically correct values indicated by Eqs. 3 have some properties that can be

profitably studied before Eqs. 3 are thrown into the form of Eqs. 5 for use in the method of iteration. For a solution, Eqs. 3 must conform to the following: There must be as many equations as unknowns; the equations must be independent of each other; and possible conditions must be represented. Solving Eqs. 3 in determinant form and representing expressions for numerator and denominator by $N$ and $D$, respectively, $x_a = \frac{N_a}{D}$, etc. If $D = 0$, the left-hand side of the equations are functionally related. If $N = 0$ also, the right-hand sides are related functionally in the same manner. Under the aforementioned conditions the equations are dependent on each other and by themselves will not lead to a solution. If $D = 0$, $N \neq 0$ the equations do not represent possible conditions.

In using the method of iteration once a definite trend has been established, by plotting known values against cycles the range can be extended graphically. If this is carefully done and good judgment is exercised the method of iteration itself will be considerably shortened.

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DISCUSSIONS

# EXPANSION OF CONCRETE THROUGH REACTION BETWEEN CEMENT AND AGGREGATE

## Discussion

BY BAILEY TREMPER, ESQ.

BAILEY TREMPER,[8] ESQ. (by letter).[8a]—The physical consequences that may result from chemical reactions between high-alkali cements and certain mineral constituents of aggregates have been presented convincingly in this paper. The author's disclosures serve to focus attention on concrete in other localities where the outward appearance of internal expansion is similar to that illustrated in Fig. 1.

In 1935, H. S. Mattimore and G. A. Rahn, Jr.,[9] Assoc. Members, Am. Soc. C. E., reported on the condition of some experimental concrete test walls that had been exposed to weather in Pennsylvania and were then 4½ years old. Two cements and two fine aggregates were used in the concrete. The alkali content of the cements was as follows:

| Chemical | Brand A | Brand B |
|---|---|---|
| $Na_2O$ | 0.43 | 0.24 |
| $K_2O$ | 0.57 | 0.37 |
| | 1.00 | 0.61 |

There was not much difference in the composition of the two cements in other respects.

According to the scheme of rating used, all of the specimens made with brand B cement were rated as 70% intact or better. The ratings for specimens made with brand A cement were from 0 to 80%, averaging 51% and 75%, respectively, for the two sands.

In the State of Washington there are a few examples of concrete exhibiting patterns of open cracks while other concrete made with aggregates from the

NOTE.—This paper by Thomas E. Stanton, M. Am. Soc. C. E., was published in December, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1941, by R. W. Carlson, Assoc. M. Am. Soc. C. E.

[8] Materials Engr., Dept. of Highways, State of Washington, Olympia, Wash.

[8a] Received by the Secretary February 4, 1941.

[9] "Research on Concrete Disintegration," by H. S. Mattimore and G. A. Rahn, Jr., *Proceedings*, A. S. T. M., Vol. 35, 1935, p. 410.

same sources is in excellent condition. The defects are connected definitely with the source, and probably with the alkali content, of the cements. In one instance the aggregate used in the defective concrete was almost entirely igneous in origin.

These occurrences give rise to the uncomfortable suspicion that deleterious expansion may result from the use of high-alkali cement with a variety of aggregates that is very much more extensive than the sedimentary group described by the author.

Mr. Stanton's physical test, consisting of the measurement of length changes of 1 by 10-in. mortar bars, seems to have served very well in distinguishing between good and bad combinations of cement and aggregate in the locality in which he has worked. It is not known, however, that the test will serve equally well with other types of aggregate. It is unfortunate, moreover, that several months at least are required to obtain significant results.

These considerations point to the desirability of knowing more concerning the chemical nature of the reactions involved. When this information is available it should be possible to devise a more rapid method of eliminating suspicion from satisfactory aggregates.

Mr. Stanton points to two types of chemical reactions as possible causes of expansion. The first is a reaction between sodium hydroxide and the magnesium carbonate occurring in the siliceous magnesian limestone. The second is a reaction between the alkalis and silica in the aggregate.

The first reaction does not appear to the writer to be the basic cause of expansion for the following reasons:

(1) Expansions were observed with aggregates containing magnesia in relatively small amounts.

(2) Although the paper does not report the separate percentages of sodium oxide and potassium oxide in the cements involved, it is logical to assume that they were present in varying ratios. The observed expansions were in good agreement with the total alkali content of the cements. This indicates that sodium and potassium compounds were equally active. The latter does not give the theoretical increase in volume indicated for the sodium-magnesium reaction.

(3) Calcium hydroxide, always present in relatively large amounts in hardened concrete, should prevent the formation of sodium or potassium carbonates.

The reaction involving silica in the aggregate, as illustrated in Table 3, seems to be a more valid explanation of the expansion. Rocks that show puzzolanic action when finely ground are generally soluble in part in alkali solutions. Alumina as well as silica may be dissolved. If such a reaction is the cause of expansion in concrete, then one might expect rocks such as rhyolite and andesite, particularly when formed by explosive volcanic action, to be troublesome with high-alkali cements.

In the consideration of such a reaction, the effect of dissolved calcium hydroxide cannot be ignored. Nearly all calcium salts are less soluble than

the corresponding sodium or potassium salts. Therefore, the final products would be expected to be calcium silicates or aluminates with the alkalis converted back into the hydroxide form and available for additional attack upon the minerals. If the alkalis play a catalytic rôle of this nature, their potency in relatively small percentages may be more readily comprehended.

Reactions of this kind would continue in the presence of moisture until stopped by exhaustion of the active minerals, exhaustion of the calcium hydroxide, the accumulation of high pressures within the concrete, or the formation of reaction products in amounts sufficient to form a barrier against the migration of the reacting constituents.

If reactions of this kind are the true cause of expansion, further studies must take into consideration the change in solubility of calcium hydroxide in varying concentrations of sodium and potassium hydroxides, the effect of temperature on solubilities, the density and permeability of the concrete, and its capability to resist the pressure built up by the reaction products. It is hoped that Mr. Stanton's studies will progress to the point that the chemical phases can be discussed more fully.

It is worthy of emphasis that it was only with aggregates from a relatively small area in the State of California that deleterious effects were found through the use of high-alkali cements. D. G. Miller and P. W. Manson [10] have determined the alkali contents of 106 commercial cements from 74 mills. Their analyses, therefore, should represent a typical cross section of cement as produced in the United States. Of the 106 cements, more than one half contained alkalis (computed as $Na_2O$) of more than 0.6%, the maximum set by Mr. Stanton for freedom from deleterious expansion. If an aggregate has been used previously with cements from several mills, it is probable that in at least one instance the cement was high in alkali. Examination of concrete in service, then, should form a basis for estimating the probable effects on proposed construction of that aggregate with high-alkali cement even though the alkali content of previous cements is not known.

Corrections for *Transactions*: In Figs. 5 and 6 change "Limestone" to "Mineral" wherever it occurs; in the caption to Fig. 6 change "partial" to "particle"; and, in Table 5, Appendix III, heading of last column, change "experience" to "expansion."

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[10] "Tests of 106 Commercial Cements for Sulfate Resistance," by D. G. Miller and P. W. Manson, *Proceedings*, A. S. T. M., Vol. 40 (1940).

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DISCUSSIONS

# CONCRETE IN SEA WATER: A REVISED VIEWPOINT NEEDED

## Discussion

By Thomas E. Stanton, M. Am. Soc. C. E.

Thomas E. Stanton,[11] M. Am. Soc. C. E. (by letter).[11a]—Mr. Hadley is to be complimented for his presentation of the results of his extended observations of concrete marine structures along the Pacific Coast of the United States and Canada and for his emphasis on density and uniformity. The writer is in accord with Mr. Hadley's conclusions that if, in concrete exposed to sea water, every care is exercised to use sound materials, to obtain uniform high density and impermeability of concrete, and to protect reinforcement from corrosion, there need be little fear of subsequent disintegration due to sea-water attack, at least in so far as cements similar to those under observation are concerned, which probably include all of the standard cements manufactured on the Pacific Coast. That this is true everywhere, however, can be ascertained only through similar observations conducted throughout the world and with all brands of cement. Certainly the literature on cement and concrete is replete with "alleged" instances of salt-water disintegration.

The writer checks Mr. Hadley's observation of numerous concrete structures, subject to sea-water attack along the Pacific Coast, which are at least 25 years old and apparently as sound today as when constructed. This observation, combined with confirmatory laboratory tests, was the basis for the writer's conclusions, published[12] in 1938, that density and uniformity are the first considerations for a durable concrete, assuming that sound ingredients are used.

It was definitely observed, however, based on laboratory tests, that porous mortars of unquestionably sound fine aggregates, but with a high $C_3A$ portland cement, are non-durable[13] when subjected to attack by sea water of normal

Note.—This paper by Homer M. Hadley, Assoc. M. Am. Soc. C. E., was published in January, 1941, *Proceedings*.

[11] Materials and Research Engr., State Div. of Highways, Sacramento, Calif.

[11a] Received by the Secretary January 27, 1941.

[12] "Resistance of Cements to Attack by Sea Water and by Alkali Soils," by Thomas E. Stanton, *Journal*, A. C. I., Vol. 9, March–April, 1938, pp. 433 *et seq.*

[13] *Loc. cit.*, supplement, September, 1938, p. 464–7.

concentration, but that mortars using a low $C_3A$ cement are highly resistant even in 1 : 3 ungraded Ottawa sand mortars.

The conclusion was reached, therefore, that, for insurance against disintegration due to sea-water attack in the case of porous concrete, at least a moderate sulfate-resistant (8% or less of $C_3A$) cement might well be specified, particularly if such a cement can be secured without paying a premium. It is freely conceded, however, that "The vitally important matter for concrete used in marine work is to have dense, impermeable concrete made of sound materials with adequate cover over reinforcement" and that, if these conditions are met, little concern need be felt relative to the cement component, provided the cement is sound in every respect and of the general composition of those manufactured by Pacific Coast mills. The exception to the foregoing conclusions, of course, is the condition described by the writer in the paper to which Mr. Hadley refers.[7]

Correction for *Transactions:* Add to footnote 7, "See also *Proceedings*, Am. Soc. C. E., December, 1940, p. 1781."

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[7] "Influence of Cement and Aggregate on Concrete Expansion," by Thomas E. Stanton, *Engineering News-Record*, February 1, 1940, Fig. 2, pp. 59–61; see also *Proceedings*, Am. Soc. C. E., December, 1940, p. 1781.

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DISCUSSIONS

# TRANSATLANTIC SEAPLANE BASE, BALTIMORE, MARYLAND

## Discussion

By LeRoy L. Odell, Esq.

LeRoy L. Odell,[5] Esq. (by letter).[5a]—The basic conceptions from which this design developed became established in the writer's mind in 1933 in an effort to perfect an economical form of hangar that would possess the greatest possibility of future utility, having in mind the recurring step-by-step increases in the size of aircraft, which still continue to render obsolescent most earlier hangar buildings. An indication of such a need will become apparent from the fact that it is believed that first-line land aircraft in domestic commercial air transport services will have capacities approximating 80 passengers by 1950, in comparison with 33-passenger ships—today's largest. Mr. Pagon's reduction of the number of columns to four, enforced by foundation conditions, is a definite improvement upon the original plans from which his design was developed, which had a cantilever roof structure over two rows of irregularly spaced columns, and doors on all four sides.

Another matter of interest in the design features of the Baltimore hangar is the fact that wind loadings are assumed as being negative (uplift) upon the roof. Design practices that contemplate the resolution of wind forces into components parallel and normal (pressure) to roof lines are erroneous. Wind tunnel experiments demonstrate clearly that such method is incorrect and, as early as 1929, the writer, as chief airport engineer for Pan American Airways, began the control of hangar design upon the basis that wind loadings upon roofs were negative (uplift). No Pan American Airways' hangar has yet suffered wind storm damage (except that caused by flying debris from other sources) notwithstanding the fact that several such hangars have passed through hurricanes in which literally thousands of lives have been lost.

Architects and engineers responsible for the structural design of buildings would do well to look carefully into this factor in their design problems, since thereby substantial savings, together with added safety, become assured.

Note.—This paper by W. Watters Pagon, M. Am. Soc. C. E., was published in October, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: February, 1941, by Karl Terzaghi, M. Am. Soc. C. E.

[5] Engr., New York, N. Y.

[5a] Received by the Secretary February 7, 1941.